Università degli Studi di Torino
Scuola di Dottorato

UNIVERSITÀ DEGLI STUDI DI TORINO

Topics on Theoretical and Computational Aspects of String Theory and Their Phenomenological Implications

Riccardo Finotello

Università degli Studi di Torino
Scuola di Dottorato

Dottorato in Fisica ed Astrofisica

Topics on Theoretical and Computational Aspects of String Theory and Their Phenomenological Implications

Riccardo Finotello

Advisor: Igor Pesando

Abstract

Acknowledgements

Riccardo Finotello

Table of Contents

Outline 11

I Conformal Symmetry and Geometry of the Wordlsheet 13

1 Introduction 15

1.1 Properties of String Theory and Conformal Symmetry 15

1.1.1 Action Principle 15

1.1.2 Conformal Invariance 16

1.2 Extra Dimensions and Compactification 17

1.3 D-branes and Open Strings 17

2 D-branes Intersecting at Angles 17

3 Fermions With Boundary Defects 17

II Cosmology and Time Dependent Divergences 19

4 Introduction 21

III Deep Learning the Geometry of String Theory 23

5 Introduction 25

References 25

Outline

Part I

Conformal Symmetry and Geometry of the Wordlsheet

1 Introduction

In this first part we focus on aspects of string theory directly connected with its worldsheet description and symmetries. The underlying idea is to build technical tools to address the study of viable phenomenological models in this framework. In fact the construction of realistic string models of particle physics is the key to better understanding the nature of a theory of everything such as string theory.

As a first test of validity, the string theory should properly extend the known Standard Model (SM) of particle physics, which is arguably one of the most experimentally backed theoretical frameworks in modern physics. In particular its description in terms of fundamental strings should be able to include a gauge algebra locally isomorphic to that of

$$\mathrm{SU}(3)_{\mathrm{C}} \otimes \mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{U}(1)_{\mathrm{Y}} \tag{1.1}$$

in order to reproduce known results. For instance, string theory could provide a unified framework by predicting the existence of a larger gauge group containing the SM as a subset. In what follows we deal with the definition of mathematical tools to compute amplitudes to be used in phenomenological calculations related to the study of particles in string theory.

1.1 Properties of String Theory and Conformal Symmetry

Strings are extended one-dimensional objects. They are curves in space parametrized by a coordinate $\sigma \in [0, \ell]$. Propagating in D-dimensional spacetime they span a two-dimensional surface, the *worldsheet*, described by the position of the string at given values of σ at a time τ, i.e. $X^{\mu}(\tau, \sigma)$ where $\mu = 0, 1, \ldots, D-1$ indexes the coordinates.

1.1.1 Action Principle

As the action of a point particle is proportional to the length of its trajectory (its *worldline*), the same object for string is proportional to the area of the worldsheet in the original formulation by Nambu and Goto. The solutions of the classical equations of motion (E.O.M.) are therefore strings spanning a worldsheet of extremal area. While Nambu and Goto's formulation is fairly direct in its definition, it usually best to work with Polyakov's action [1]

$$S_P[\gamma, X] = -\frac{T}{2} \int_{-\infty}^{+\infty} \mathrm{d}\tau \int_0^{\ell} \mathrm{d}\sigma \sqrt{-\det \gamma}\, \gamma^{\alpha\beta}\, \partial_\alpha X^{\mu}(\tau, \sigma)\, \partial_\beta X^{\nu}(\tau, \sigma)\, \eta_{\mu\nu}. \tag{1.2}$$

In this formulation $\gamma_{\alpha\beta}$ is the worldsheet metric with Lorentzian signature $(-, +)$. As there are no derivatives of $\gamma_{\alpha\beta}$, its E.O.M. is a constraint ensuring the equivalence of Polyakov's and Nambu and Goto's formulations. In fact

$$\frac{\delta S_P[\gamma, X]}{\delta \gamma^{\alpha\beta}} = \partial_\alpha X \cdot \partial_\beta X - \frac{1}{2} \gamma_{\alpha\beta}\, \gamma^{\lambda\rho}\, \partial_\lambda X \cdot \partial_\rho X = 0 \tag{1.3}$$

implies

$$S_P[\gamma, X]\Big|_{\frac{\delta S_P[\gamma,X]}{\delta\gamma^{\alpha\beta}}=0} = -T\int_{-\infty}^{+\infty} d\tau \int_0^{\sigma} d\sigma\,\sqrt{\dot{X}\cdot\dot{X} - X'\cdot X'} = S_{NG}[X], \tag{1.4}$$

where $S_{NG}[X]$ is the Nambu–Goto action for the classical string.

The symmetries of $S_P[\gamma, X]$ are the keys to the success of the string theory framework [2]. Specifically (1.2) displays symmetries under:

- D-dimensional Poincaré invariance

$$\begin{aligned} X'^{\mu}(\tau,\sigma) &= \Lambda^{\mu}{}_{\nu}\, X^{\mu}(\tau,\sigma) + c^{\nu}, \\ \gamma'_{\alpha\beta}(\tau,\sigma) &= \gamma_{\alpha\beta}(\tau,\sigma) \end{aligned} \tag{1.5}$$

  where $\Lambda \in \mathrm{SO}(1, D-1)$ and $c \in \mathbb{R}^D$,

- 2-dimensional diffeomorphism invariance

$$\begin{aligned} X'^{\mu}(\tau',\sigma') &= X^{\mu}(\tau,\sigma) \\ \gamma'_{\alpha\beta}(\tau',\sigma') &= \frac{\partial\sigma'^{\lambda}}{\partial\sigma^{\alpha}}\frac{\partial\sigma'^{\rho}}{\partial\sigma^{\beta}}\gamma_{\lambda\rho}(\tau,\sigma) \end{aligned} \tag{1.6}$$

  where $\sigma^0 = \tau$ and $\sigma^1 = \sigma$,

- Weyl invariance

$$\begin{aligned} X'^{\mu}(\tau',\sigma') &= X^{\mu}(\tau,\sigma) \\ \gamma'_{\alpha\beta}(\tau,\sigma) &= e^{2\omega(\tau,\sigma)}\,\gamma_{\alpha\beta}(\tau,\sigma) \end{aligned} \tag{1.7}$$

  for arbitrary $\omega(\tau,\sigma)$.

Notice that the last is not a symmetry of the Nambu–Goto action and it only appears in Polyakov's formulation of the action.

1.1.2 Conformal Invariance

The definition of the 2-dimensional stress-energy tensor is a direct consequence of (1.3) [3]. In fact the classical constraint on the tensor is simply

$$T_{\alpha\beta} = -\frac{2\pi}{\sqrt{-\det\gamma}}\frac{\delta S_P[\gamma,X]}{\delta\gamma^{\alpha\beta}} = 0. \tag{1.8}$$

While its conservation $\nabla^{\alpha}T_{\alpha\beta} = 0$ is somewhat trivial, Weyl invariance also ensures the tracelessness of the tensor

$$\mathrm{tr}\,T = T^{\alpha}{}_{\alpha} = 0. \tag{1.9}$$

In other words, the $(1+1)$-dimensional theory of massless scalars X^{μ} in (1.2) is *conformally invariant* (for review and details see [4], [5]).

Finally we can set $\gamma_{\alpha\beta}(\tau,\sigma) = e^{\phi(\tau,\sigma)}\,\eta_{\alpha\beta}$, known as *conformal gauge* where $\eta_{\alpha\beta} = \mathrm{diag}(-1,1)$, using the invariances of the action. This gauge choice is however preserved by the residual *pseudo-conformal* transformations

$$\tau \pm \sigma = \sigma_\pm \mapsto f_\pm(\sigma_\pm), \tag{1.10}$$

where $f_\pm(\xi)$ are arbitrary functions. It is natural to introduce a Wick rotation $\tau_E = i\tau$ and the complex coordinates $\xi = \tau_E + i\sigma$ and $\overline{\xi} = \xi^*$. In these terms, the tracelessness of the stress-energy tensor translates to

$$T_{z\overline{z}} = 0, \tag{1.11}$$

while its conservation $\partial^\alpha T_{\alpha\beta} = 0$ becomes:[1]

$$\overline{\partial} T_{\xi\xi}(\xi,\overline{\xi}) = \partial \overline{T}_{\overline{\xi}\overline{\xi}}(\xi,\overline{\xi}) = 0. \tag{1.12}$$

The last equation finally implies

$$T_{\xi\xi}(\xi,\overline{\xi}) = T_{\xi\xi}(\xi) = T(\xi), \qquad \overline{T}_{\overline{\xi}\overline{\xi}}(\xi,\overline{\xi}) = \overline{T}_{\overline{\xi}\overline{\xi}}(\overline{\xi}) = \overline{T}(\overline{\xi}), \tag{1.13}$$

which are respectively the holomorphic and the anti-holomorphic components of the 2-dimensional stress energy tensor.

The previous properties define what is known as a 2-dimensional *conformal field theory* (CFT). Ordinary tensor fields

$$\phi_{\omega,\overline{\omega}}(\xi,\overline{\xi}) = \phi_{\underbrace{\xi\ldots\xi}_{\omega\ \text{times}}\,\underbrace{\overline{\xi}\ldots\overline{\xi}}_{\overline{\omega}\ \text{times}}}(\xi,\overline{\xi})\,(\mathrm{d}\xi)^{\omega}\,\left(\mathrm{d}\overline{\xi}\right)^{\overline{\omega}} \tag{1.14}$$

can be classified according to their weights ω and $\overline{\omega}$. In fact a transformation $\xi \mapsto \chi(\xi)$ and $\overline{\xi} \mapsto \overline{\chi}(\overline{\xi})$ maps the conformal fields to

$$\phi_{\omega,\overline{\omega}}(\chi,\overline{\chi}) = \left(\frac{\mathrm{d}\chi}{\mathrm{d}\xi}\right)^{\omega}\left(\frac{\mathrm{d}\overline{\chi}}{\mathrm{d}\overline{\xi}}\right)^{\overline{\omega}}\phi_{\omega,\overline{\omega}}(\xi,\overline{\xi}). \tag{1.15}$$

1.2 Extra Dimensions and Compactification

1.3 D-branes and Open Strings

2 D-branes Intersecting at Angles

3 Fermions With Boundary Defects

[1] Since we fix $\gamma_{\alpha\beta}(\tau,\sigma) \propto \eta_{\alpha\beta}$ we do not need to account for the components of the connection and we can replace the covariant derivative ∇^α with a standard derivative ∂^α.

Part II

Cosmology and Time Dependent Divergences

4 Introduction

Part III

Deep Learning the Geometry of String Theory

5 Introduction

References

[1] A. Polyakov, 'Quantum geometry of bosonic strings', *Physics Letters B*, vol. 103, no. 3, pp. 207–210, Jul. 1981, ISSN: 03702693. DOI: 10.1016/0370-2693(81)90743-7. [Online]. Available: 10.1016/0370-2693(81)90743-7 (visited on 31/08/2020).

[2] J. Polchinski, *String Theory. An Introduction to the Bosonic String.* 1st ed., 2 vols. Cambridge University Press, 13th Oct. 1998, vol. 1, ISBN: 978-0-521-67227-6 978-0-521-63303-1 978-0-511-81607-9. DOI: 10.1017/CBO9780511816079; http://web.archive.org/web/20200831133025/https://www.cambridge.org/core/books/string-theory/30409AF2BDE27D53E275FDA395AB667A. [Online]. Available: https://doi.org/10.1017/CBO9780511816079 (visited on 31/08/2020).

[3] M. B. Green, J. Schwarz and E. Witten, *Superstring Theory. Introduction.* 2 vols., Cambridge Monographs on Mathematical Physics. Jul. 1988, vol. 1, ISBN: 978-0-521-35752-4.

[4] D. Friedan, E. J. Martinec and S. H. Shenker, 'Conformal invariance, supersymmetry and string theory', *Nuclear Physics B*, vol. 271, no. PRINT-86-0024 (CHICAGO), EFI-85-89-CHICAGO, pp. 93–165, 1986. DOI: 10.1016/0550-3213(86)90356-1. [Online]. Available: 10.1016/0550-3213(86)90356-1.

[5] P. Di Francesco, P. Mathieu and D. Senechal, *Conformal Field Theory*, ser. Graduate Texts in Contemporary Physics. New York: Springer-Verlag, 1997, ISBN: 978-0-387-94785-3 978-1-4612-7475-9. DOI: 10.1007/978-1-4612-2256-9.