

D-branes and Deep Learning

Theoretical and Computational Aspects in String Theory

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Conformal Symmetry and Geometry of the Worldsheet

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Orbifolds and Cosmological Toy Models

Null Boost Orbifold

Deep Learning the Geometry of String Theory

Machine Learning and Deep Learning

Machine Learning for String Theory

AI Implementations for Geometry and Strings

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Action Principle and Conformal Symmetry

Polyakov's Action

$$S_P[\gamma, X, \psi] = -\frac{1}{4\pi} \int_{-\infty}^{+\infty} d\tau \int_0^\ell d\sigma \sqrt{-\det \gamma} \gamma^{\alpha\beta} \left(\frac{2}{\alpha'} \partial_\alpha X^\mu \partial_\beta X^\nu + \psi^\mu \rho_\alpha \partial_\beta \psi^\nu \right) \eta_{\mu\nu}$$

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Symmetries:

Poincaré transf.: $X'^\mu = \Lambda^\mu_\nu X^\nu + c^\mu$
2D diff.: $\gamma'_{\alpha\beta} = (J^{-1})_{\alpha\beta}^{\lambda\rho} \gamma_{\lambda\rho}$
Weyl transf.: $\gamma'_{\alpha\beta} = e^{2\omega} \gamma_{\alpha\beta}$

Conformal symmetry:

vanishing stress-energy tensor: $\mathcal{T}_{\alpha\beta} = 0$
traceless stress-energy tensor: $\text{tr } \mathcal{T} = 0$
conformal gauge: $\gamma_{\alpha\beta} = e^\Phi \eta_{\alpha\beta}$

Action Principle and Conformal Symmetry

Superstrings in D dimensions $\longrightarrow$ *Virasoro algebra* (central extension of de Witt's algebra):

$$\mathcal{T}(z) = -\frac{1}{\alpha'} \partial X(z) \cdot \partial X(z) - \frac{1}{2} \psi(z) \cdot \partial \psi(z) \quad \Rightarrow \quad c = \frac{3}{2} D$$

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$(\lambda, 0) / (1 - \lambda, 0)$ **Ghost System**

Introduce anti-commuting (b, c) and commuting (β, γ) conformal fields:

$$S_{\text{ghost}}[b, c, \beta, \gamma] = \frac{1}{2\pi} \iint dz d\bar{z} (b(z) \bar{\partial} c(z) + \beta(z) \bar{\partial} \gamma(z))$$

where $\lambda_b = 2$ and $\lambda_c = -1$, and $\lambda_\beta = \frac{3}{2}$ and $\lambda_\gamma = -\frac{1}{2}$.

[Friedan, Martinec, Shenker (1986)]

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Consequence:

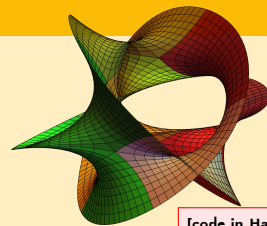
$$c_{\text{full}} = c + c_{\text{ghost}} = 0 \quad \Leftrightarrow \quad D = 10.$$

Extra Dimensions and Compactification

Compactification

$$\mathcal{M}^{1,9} = \mathcal{M}^{1,3} \otimes \mathcal{X}_6$$

- $\mathcal{X}_6$ is a **compact** manifold
- $N = 1$ **supersymmetry** preserved in 4D
- contains algebra of $SU(3) \otimes SU(2) \otimes U(1)$



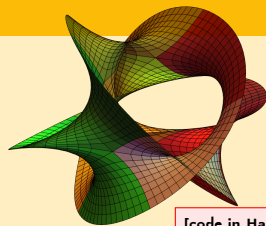
[code in Hanson (1994)]

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Calabi–Yau manifolds (M, g) such that:

- $\dim_{\mathbb{C}} M = m$
- $\text{Hol}(g) \subseteq SU(m)$
- $\text{Ric}(g) \equiv 0$ (equiv. $c_1(M) \equiv 0$)

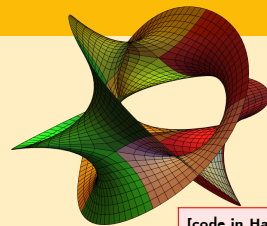
[Calabi (1957), Yau (1977), Candelas et al. (1985)]

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[Calabi (1957), Yau (1977), Candelas et al. (1985)]

Characterised by **Hodge numbers**

$$h^{r,s} = \dim_{\mathbb{C}} H_{\bar{\partial}}^{r,s}(M, \mathbb{C})$$

(no. of harmonic (r,s) -forms).

D-branes and Open Strings

Polyakov's action naturally introduces **Neumann b.c.**:

$$\partial_\sigma X(\tau, \sigma) \Big|_{\sigma=0}^{\sigma=\ell} = 0$$

satisfied by **open and closed strings** in D dim. s.t. $\square X = 0 \Rightarrow X(z, \bar{z}) = X(z) + \overline{X}(\bar{z})$.

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T-duality

Consider **closed strings** on $\mathcal{M}^{1,D-1} = \mathcal{M}^{1,D-2} \otimes S^1(R)$:

$$\begin{aligned} \begin{cases} \alpha_0^{D-1} &= \frac{1}{\sqrt{2\alpha'}} \left(n \frac{\alpha'}{R} + mR \right) \\ \tilde{\alpha}_0^{D-1} &= \frac{1}{\sqrt{2\alpha'}} \left(n \frac{\alpha'}{R} - mR \right) \end{cases} \Rightarrow M^2 = -p^\mu p_\mu = \frac{2}{\alpha'} (\alpha_0^{D-1})^2 + \frac{4}{\alpha'} (N + a) \\ &= \frac{2}{\alpha'} (\tilde{\alpha}_0^{D-1})^2 + \frac{4}{\alpha'} (\tilde{N} + a) \end{aligned}$$

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T-duality

Dirichlet b.c. consequence of **T-duality** on p directions:

$$\bar{X}(z) \mapsto -\bar{X}(z) \quad \Rightarrow \quad \partial_\sigma X^i(\tau, \sigma) \Big|_{\sigma=0}^{\sigma=\ell} = 0 \quad \xrightarrow{T\text{-duality}} \quad \partial_\tau \tilde{X}^i(\tau, \sigma) \Big|_{\sigma=0}^{\sigma=\ell} = 0$$

thus **open strings** can be **constrained** to $D(D - p - 1)$ -branes.

[Polchinski (1995, 1996)]

D-branes and Open Strings

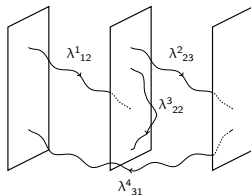
Introducing Dp -branes breaks $\text{ISO}(1, D-1) \rightarrow \text{ISO}(1, p) \otimes \text{SO}(D-1-p)$:

$$\mathcal{A}^\mu \rightarrow (\mathcal{A}^A, \mathcal{A}^a) \quad \Rightarrow \quad \text{U}(1) \text{ theory in } p+1 \text{ dimensions (and scalars)}$$

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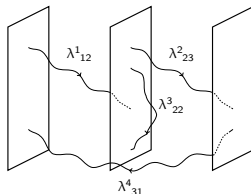
[Chan, Paton (1969)]

$$|n; r\rangle = \sum_{i,j=1}^N |n; i, j\rangle \lambda^r_{ij} \Rightarrow \text{U}(N)$$

D-branes and Open Strings

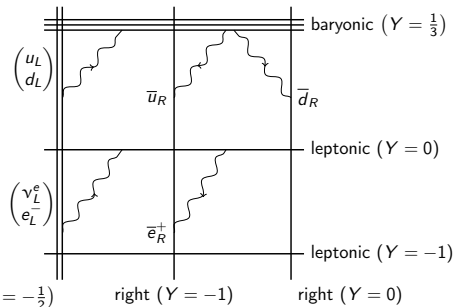
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Intersecting D-branes

Consider 3 intersecting *D6*-branes filling $\mathcal{M}^{1,3}$ and embedded in $\mathbb{R}^6$

Twist Fields Correlators

$$\left\langle \prod_{t=1}^{N_B} \sigma_{M_{(t)}}(x_{(t)}) \right\rangle = \mathcal{N} \left(\{x_{(t)}, M_{(t)}\}_{1 \leq t \leq N_B} \right) e^{-S_{E(\text{cl})} \left(\{x_{(t)}, M_{(t)}\}_{1 \leq t \leq N_B} \right)}$$

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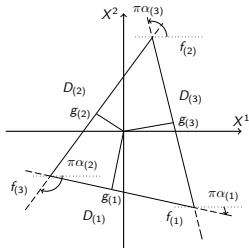
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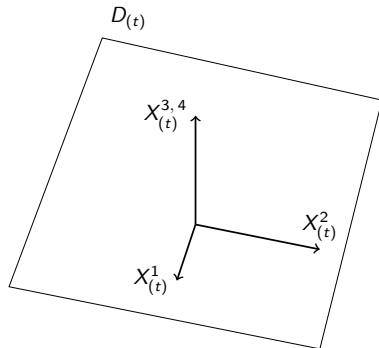
D-branes in **factorised** internal space:

- **embedded as lines** in $\mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2$
- **relative rotations** are $SO(2) \simeq U(1)$ elements
- $S_E^{(\text{cl})} \left(\{x(t), M(t)\}_{1 \leq t \leq N_B} \right) \sim \text{Area} \left(\{f(t), R(t)\}_{1 \leq t \leq N_B} \right)$

[Cremades, Ibanez, Marchesano (2003); Pesando (2012)]

SO(4) Rotations

Consider $\mathbb{R}^4 \times \mathbb{R}^2$ (focus on $\mathbb{R}^4$):



$$(X_{(t)})^I = (R_{(t)})^I_J X^J - g_{(t)}^I \in \mathbb{R}^4$$

where

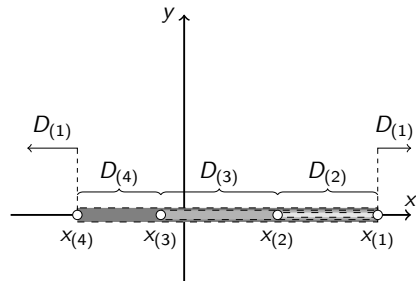
$$R_{(t)} \in \frac{\text{SO}(4)}{\text{S}(\text{O}(2) \times \text{O}(2))}$$

that is

$$[R_{(t)}] = \{R_{(t)} \sim \mathcal{O}_{(t)} R_{(t)}\}$$

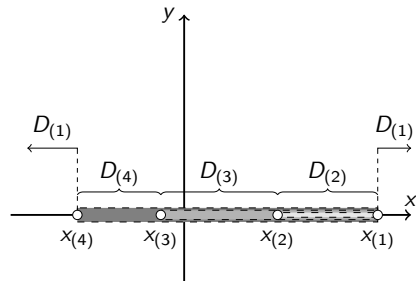
Boundary Conditions and Open Strings

- $u = x + iy = e^{\tau_e + i\sigma}$ and $\bar{u} = u^*$
- $x_{(t)} < x_{(t-1)}$ **worldsheet intersection points**



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- $x_{(t)} < x_{(t-1)}$ **worldsheet intersection points**



Branch Cuts and Discontinuities for $x \in D_{(t)}$

$$\begin{cases} \partial_u X(x + i0^+) = U_{(t)} \cdot \partial_{\bar{u}} \bar{X}(x - i0^+) = \left[R_{(t)}^{-1} \cdot (\sigma_3 \otimes \mathbb{1}_2) \cdot R_{(t)} \right] \cdot \partial_{\bar{u}} \bar{X}(x - i0^+) \\ X(x_{(t)}, x_{(t)}) = f_{(t)} \end{cases}$$

Doubling Trick and Spinor Representation

Doubling Trick

$$\partial_z \mathcal{X}(z) = \begin{cases} \partial_u X(u) & \text{if } z \in \mathcal{H}_{>}^{(\bar{t})} \\ U_{(\bar{t})} \partial_{\bar{u}} \bar{X}(\bar{u}) & \text{if } z \in \mathcal{H}_{<}^{(\bar{t})} \end{cases} \Rightarrow \begin{aligned} \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i} \delta_+) &= \mathcal{U}_{(t, t+1)} \partial_z \mathcal{X}(x_{(t)} + \delta_+), \\ \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i} \delta_-) &= \tilde{\mathcal{U}}_{(t, t+1)} \partial_z \mathcal{X}(x_{(t)} + \delta_-), \end{aligned}$$

where $\mathcal{H}_{\gtrless}^{(t)} = \{z \in \mathbb{C} \mid \text{Im } z \gtrless 0 \text{ or } z \in D_{(t)}\}$ and $\delta_{\pm} = \eta \pm i0^+$.

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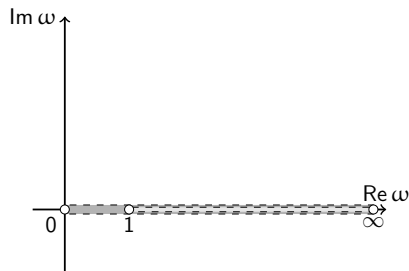
Use Pauli matrices $\tau = (i \mathbb{1}_2, \vec{\sigma})$:

$$\partial_z \mathcal{X}_{(s)}(z) = \partial_z \mathcal{X}'(z) \tau_I \Rightarrow \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i} \delta_{\pm}) = \overset{(\sim)}{\mathcal{L}}_{(t, t+1)} \partial_z \mathcal{X}(x_{(t)} + \delta_{\pm}) \overset{(\sim)}{\mathcal{R}}_{(t, t+1)}$$

where

$$\overset{(\sim)}{\mathcal{L}}_{(t, t+1)} \in \text{SU}(2)_L \quad \text{and} \quad \overset{(\sim)}{\mathcal{R}}_{(t, t+1)} \in \text{SU}(2)_R$$

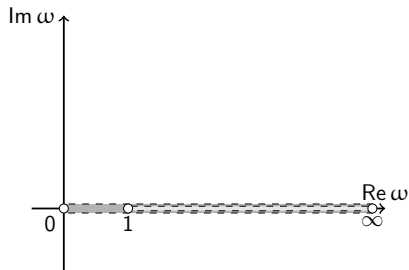
Hypergeometric Basis



Sum over all contributions:

$$\partial_z \mathcal{X}(z) = \frac{\partial \omega_z}{\partial z} \sum_{l, r=-\infty}^{+\infty} c_{lr} (-\omega_z)^{A_{lr}} (1 - \omega_z)^{B_{lr}} \\ \times B_{0,l}^{(L)}(\omega_z) \left(B_{0,r}^{(R)}(\omega_z) \right)^T$$

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Basis of Solutions

$$B_{0,n}(\omega_z) = \begin{pmatrix} 1 & 0 \\ 0 & K_n \end{pmatrix} \begin{pmatrix} \frac{1}{\Gamma(c_n)} {}_2F_1(a_n, b_n; c_n; \omega_z) \\ \frac{(-\omega_z)^{1-c_n}}{\Gamma(2-c_n)} {}_2F_1(a_n + 1 - c_n, b_n + 1 - c_n; 2 - c_n; \omega_z) \end{pmatrix}$$

The Solution

Sequence of the operations:

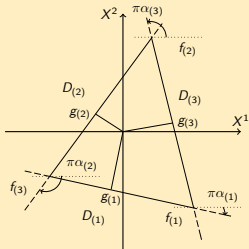
1. rotation matrix = monodromy matrix
2. contiguity relations $\Rightarrow$ independent hypergeometrics
3. finite action $\Rightarrow$ 2 solutions (no. of d.o.f. is correctly saturated)
4. boundary conditions $\Rightarrow$ fix free constants c_{lr}

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Physical Interpretation



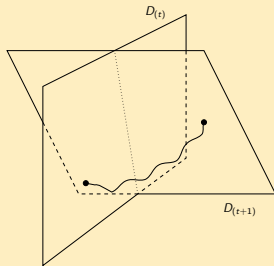
$$\begin{aligned}
 2\pi\alpha' S_{\mathbb{R}^4} \Big|_{\text{on-shell}} &= \sum_{t=1}^3 \left(\frac{1}{2} \left| g_{(t)}^\perp \right| \left| f_{(t-1)} - f_{(t)} \right| \right) \\
 &= \text{Area} \left(\{ f_{(t)} \}_{1 \leq t \leq N_B} \right)
 \end{aligned}$$

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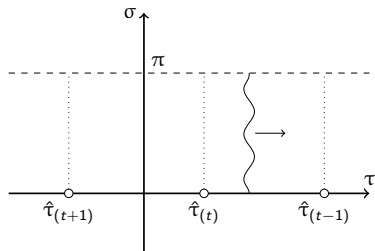
Physical Interpretation



- strings no longer confined to plane
- strings form a *small bump* from the D-brane
- classical action **larger** than factorised case

[RF, Pesando (2019)]

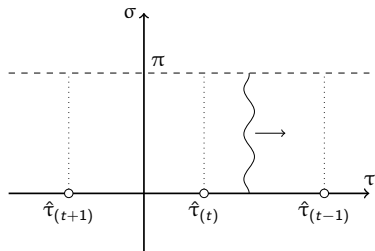
Fermions on the Strip



Action of Boundary Changing Operators

$$\begin{cases} \psi_-^i(\tau, 0) &= (R_{(t)})^i{}_j \psi_+^j(\tau, 0) \quad \text{for } \tau \in (\hat{\tau}_{(t)}, \hat{\tau}_{(t-1)}) \\ \psi_-^i(\tau, \pi) &= -\psi_+^i(\tau, \pi) \quad \text{for } \tau \in \mathbb{R} \end{cases}$$

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$$\mathcal{T}_{\pm\pm}(\xi_{\pm}) = -i \frac{T}{4} \psi_{\pm, I}^*(\xi_{\pm}) \overset{\leftrightarrow}{\partial} \psi_{\pm}^I(\xi_{\pm}) \Rightarrow \begin{cases} \dot{H}(\tau) &= 0 \\ \dot{P}(\tau) &\neq 0 \end{cases} \Leftrightarrow \tau \in (\tau_{(t)}, \tau_{(t-1)})$$

Conserved Product and Operators

Expand on a **basis of solutions**

$$\psi_{\pm}(\xi_{\pm}) = \sum_{n=-\infty}^{+\infty} b_n \psi_n(\xi_{\pm}) \quad \Rightarrow \quad \Psi(z) = \begin{cases} \psi_{E,+}(u) & \text{if } z \in \mathcal{H}_{>}^{(\bar{t})} \\ \psi_{E,-}(u) & \text{if } z \in \mathcal{H}_{<}^{(\bar{t})} \end{cases}$$

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Conserved Product and Dual Basis

$$\langle\langle {}^*\psi_n, \psi_m \rangle\rangle = 2\pi\mathcal{N} \oint \frac{dz}{2\pi i} {}^*\Psi_n {}^*\Psi_m = \delta_{n,m} \quad \Rightarrow \quad \langle\langle {}^*\Psi_n^{(*)}, \Psi^{(*)} \rangle\rangle = b_n^{(\dagger)}$$

Conserved Product and Operators

Expand on a **basis of solutions**

$$\psi_{\pm}(\xi_{\pm}) = \sum_{n=-\infty}^{+\infty} b_n \psi_n(\xi_{\pm}) \quad \Rightarrow \quad \Psi(z) = \begin{cases} \psi_{E,+}(u) & \text{if } z \in \mathcal{H}_{>}^{(\bar{t})} \\ \psi_{E,-}(u) & \text{if } z \in \mathcal{H}_{<}^{(\bar{t})} \end{cases}$$

Conserved Product and Dual Basis

$$\langle\langle {}^*\psi_n, \psi_m \rangle\rangle = 2\pi\mathcal{N} \oint \frac{dz}{2\pi i} {}^*\Psi_n {}^*\Psi_m = \delta_{n,m} \quad \Rightarrow \quad \langle\langle {}^*\Psi_n^{(*)}, \Psi^{(*)} \rangle\rangle = b_n^{(\dagger)}$$

Derive the **algebra of operators**:

$$[b_n, b_m^{\dagger}]_+ = \frac{2\mathcal{N}}{T} \langle\langle {}^*\Psi_n^*, \Psi_m^* \rangle\rangle$$

Twisted Complex Fermions

Consider $R_{(t)} = e^{i\pi\alpha_{(t)}} \in U(1)$:

$$\Psi(x_{(t)} + e^{2\pi i} \delta) = e^{i\pi\epsilon_{(t)}} \Psi(x_{(t)} + \delta)$$

where

$$\epsilon_{(t)} = \alpha_{(t+1)} - \alpha_{(t)} + \theta(\alpha_{(t)} - \alpha_{(t+1)} - 1) - \theta(\alpha_{(t+1)} - \alpha_{(t)} - 1)$$

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Basis of Solutions

$$\Psi_n(z; \{x_{(t)}\}) = \mathcal{N}_\Psi z^{-n} \prod_{t=1}^N \left(1 - \frac{z}{x_{(t)}}\right)^{n_{(t)} + \frac{\epsilon_{(t)}}{2}}$$

$$^*\Psi_n(z; \{x_{(t)}\}) = \frac{1}{2\pi\mathcal{N}\mathcal{N}_\Psi} z^{n-1} \prod_{t=1}^N \left(1 - \frac{z}{x_{(t)}}\right)^{-\tilde{n}_{(t)} + \frac{\epsilon_{(t)}}{2}}$$

Vacua

Define the **vacuum** with respect to b_n :

$$b_n \left| \{x_{(t)}\} \right\rangle = 0 \quad \text{for } n \geq 1$$

$$b_n \left| \tilde{0} \right\rangle = 0 \quad \text{for } n \geq n_{(t)} + \frac{\epsilon_{(t)}}{2} + \frac{1}{2}$$

Vacua

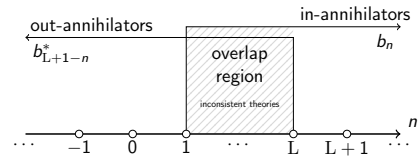
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Theories are subject to **consistency conditions**:

$$\langle \{x_{(t)}\} | \{x_{(t)}\} \rangle = 1 \quad \Rightarrow \quad L = n_{(t)} + \tilde{n}_{(t)}$$



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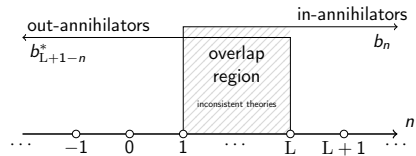
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Stress-energy Tensor and CFT Approach

Compute the OPEs leading to the **time dependent stress-energy tensor**:

$$\mathcal{T}(z) = \frac{\pi T}{2} \mathcal{N}_{\Psi}^2 \sum_{n, m=-\infty}^{+\infty} : b_n b_m^* : z^{-n-m} \left[\frac{m-n}{2} + 2 \sum_{t=1}^N \frac{n_{(t)} + \frac{\epsilon_{(t)}}{2}}{z - x_{(t)}} \right] + \frac{1}{2} \left(\sum_{t=1}^N \frac{n_{(t)} + \frac{\epsilon_{(t)}}{2}}{z - x_{(t)}} \right)^2$$

[RF, Pesando (2019)]

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Invariant Vacuum and Spin Fields

$$|\{x(t)\}\rangle = \mathcal{N}(\{x(t)\}) \mathcal{R} \left[\prod_{t=1}^M S_{(t)}(x(t)) \right] |0\rangle_{\text{SL}_2(\mathbb{R})}$$

Spin Fields Amplitudes

Equivalence with Bosonization

$$\partial_{x_{(t)}} \ln \langle \{x_{(t)}\} | \{x_{(t)}\} \rangle = \oint_{x_{(t)}} \frac{dz}{2\pi i} \frac{\langle \{x_{(t)}\} | \mathcal{T}(z) | \{x_{(t)}\} \rangle}{\langle \{x_{(t)}\} | \{x_{(t)}\} \rangle}$$

$$\Rightarrow \langle \{x_{(t)}\} | \{x_{(t)}\} \rangle = \mathcal{N}(\{\epsilon_{(t)}\}) \prod_{\substack{t=1 \\ t>u}}^N (x_{(u)} - x_{(t)})^{\left(n_{(u)} + \frac{\epsilon_{(u)}}{2}\right) \left(n_{(t)} + \frac{\epsilon_{(t)}}{2}\right)}$$

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- (semi-)phenomenological models involve **twist and spin** fields and **open strings**
- framework for **bosonic** open strings with **intersecting D-branes**
- **spin fields** as **boundary changing operators** (hidden in **defects**)
- framework for amplitudes (extension to (non) Abelian twist/spin fields?)

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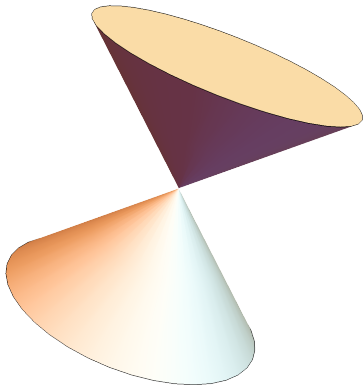
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A Few Words on a Theory of Everything

string theory = theory of everything = **nuclear forces** + **gravity**

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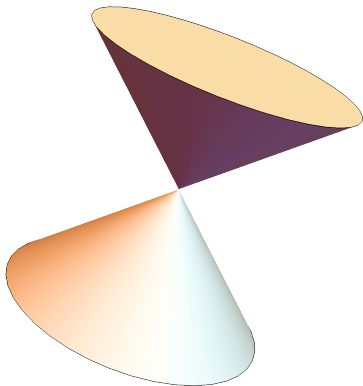


From the phenomenological point of view:

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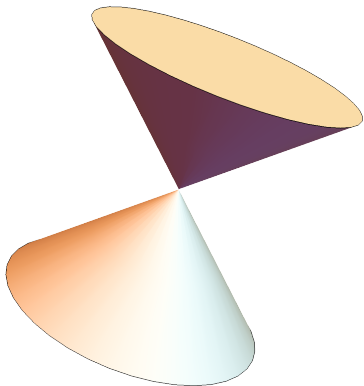


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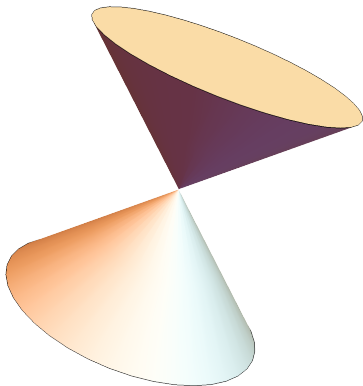


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time-dependent orbifold models

[Craps, Kutasov, Rajesh (2002); Liu, Moore, Seiberg (2002)]

Cosmological Singularities

Use **time-dependent orbifolds** to model **space-like singularities**:

divergent closed string amplitudes $\Rightarrow$ gravitational backreaction?

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Divergences

Even in simple models (e.g. NBO, more on this later) the 4 tachyons amplitude is divergent **in the open sector at tree level**:

$$A_4 \sim \int_{q \sim \infty} \frac{dq}{|q|} \mathcal{A}(q)$$

where

$$\mathcal{A}_{\text{closed}}(q) \sim q^{4-\alpha' \|\vec{p}_\perp\|^2} \quad \text{and} \quad \mathcal{A}_{\text{open}}(q) \sim q^{1-\alpha' \|\vec{p}_\perp\|^2} \text{tr}([T_1, T_2]_+ [T_3, T_4]_+)$$

Null Boost Orbifold

Start from $(x^+, x^-, x^2, \vec{x}) \in \mathcal{M}^{1,D-1}$:

$$\begin{cases} u &= x^- \\ z &= \frac{x^2}{\Delta x^-} \\ v &= x^+ - \frac{1}{2} \frac{(x^2)^2}{x^-} \end{cases} \quad \Rightarrow \quad ds^2 = -2 du dv + (\Delta u)^2 dz^2 + \delta_{ij} dx^i dx^j$$

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Killing Vector and Null Boost Orbifold

$$\kappa = -i(2\pi\Delta)J_{+2} = 2\pi\partial_z \quad \Rightarrow \quad z \sim z + 2\pi n$$

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Scalars on NBO:

$$\Phi_{\{k_+, l, \vec{k}, r\}}(u, v, z, \vec{x}) = e^{i(k_+ v + l z + \vec{k} \cdot \vec{x})} \tilde{\Phi}_{\{k_+, l, \vec{k}, r\}}(u) = \frac{e^{i(k_+ v + l z + \vec{k} \cdot \vec{x})}}{\sqrt{(2\pi)^D |2\Delta k_+ u|}} e^{-i \frac{l^2}{2\Delta^2 k_+} \frac{1}{u} + i \frac{\|\vec{k}\|^2 + r}{2k_+} u}$$

Scalar QED Interactions

Scalar-photon interactions:

$$S_{\text{sQED}}^{(\text{int})} = \int_{\Omega} d^D x \sqrt{-g} \left(-i e g^{\alpha\beta} a_{\alpha} (\phi^* \partial_{\beta} \phi - \partial_{\beta} \phi^* \phi) + e^2 g^{\alpha\beta} a_{\alpha} a_{\beta} |\phi|^2 - \frac{g^4}{4} |\phi|^4 \right)$$

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Terms involved:

$$\mathcal{I}_{\{N\}}^{[\nu]} = \int_{-\infty}^{+\infty} du |\Delta u| u^{\nu} \prod_{i=1}^N \tilde{\Phi}_{\{k_{+(i)}, l_{(i)}, \vec{k}_{(i)}, r_{(i)}\}}(u)$$

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most terms **do not converge** due to **isolated zeros** ($l_{(*)} \equiv 0$) and cannot be recovered even with a **distributional interpretation** due to the term $\propto u^{-1}$ in the exponential

String and Field Theory

So far:

- field theory presents **divergences** (see sQED)
- obvious ways to regularise (Wilson lines, higher derivative couplings, etc.) **do not work**
- divergences are **not (only) gravitational**
- **vanishing volume** in phase space responsible for the divergence

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Massive String States

$$V_M(x; k, S, \xi) =: \left(\frac{i}{\sqrt{2\alpha'}} \xi_\alpha \partial_x^2 X^\alpha(x, x) + \left(\frac{i}{\sqrt{2\alpha'}} \right)^2 S_{\alpha\beta} \partial_x X^\alpha(x, x) \partial_x X^\beta(x, x) \right) e^{ik \cdot X(x, x)}.$$

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*string theory cannot do **better than field theory** (EFT) if the latter **does not exist***

Resolution and Motivation

Introduce the generalised NBO:

$$\begin{cases} u &= x^- \\ z &= \frac{1}{2x^-} \left(\frac{x^2}{\Delta_2} + \frac{x^3}{\Delta_3} \right) \\ w &= \frac{1}{2x^-} \left(\frac{x^2}{\Delta_2} - \frac{x^3}{\Delta_3} \right) \\ v &= x^+ - \frac{1}{2x^-} \left((x^2)^2 + (x^3)^2 \right) \end{cases} \quad \Rightarrow \quad \kappa = -2\pi i (\Delta_2 J_{+2} + \Delta_3 J_{+3}) = 2\pi \partial_z$$

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No isolated zeros $\Rightarrow$ distributional Interpretation

$$\tilde{\Phi}_{\{k_+, p, l, \vec{k}, r\}}(u) = \frac{1}{2\sqrt{(2\pi)^D |\Delta_2 \Delta_3 k_+|}} \frac{1}{|u|} e^{-i \left(\frac{1}{8k_+ u} \left[\frac{(l+p)^2}{\Delta_2^2} + \frac{(l-p)^2}{\Delta_3^2} \right] - \frac{\|\vec{k}\|^2 + r}{2k_+} u \right)}$$

On the Divergences and Their Nature

- divergences are present in sQED and **open string** sector
- singularities $\Rightarrow$ **massive states** are no longer spectators
- vanishing volume (**compact orbifold directions**) $\Rightarrow$ particles “cannot escape”
- **non compact** orbifold directions $\Rightarrow$ interpretation of **amplitudes as distributions**
- issue not restricted to NBO/GNBO but also BO, null brane, etc. (it is a **general issue** connected to the geometry of the underlying space)

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*divergences are **hidden into EFT contact terms** and interactions with **string massive states**: gravity is not the only cause as the same problems are present also in gauge theories.*

[Arduino, RF, Pesando (2020)]

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The Simplest Calabi–Yau

Focus on Calabi–Yau 3-folds:

$$h^{r,s} = \dim_{\mathbb{C}} H_{\bar{\partial}}^{r,s}(M, \mathbb{C}) \quad \Rightarrow \quad \begin{cases} h^{0,0} & = h^{3,0} = 1 \\ h^{r,0} & = 0 \quad \text{if } r \neq 3 \\ h^{r,s} & = h^{3-r,3-s} \\ h^{1,1}, h^{2,1} \in \mathbb{N} \end{cases}$$

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Complete Intersection Calabi–Yau Manifolds

Intersection of hypersurfaces in

$$\mathcal{A} = \mathbb{P}^{n_1} \times \dots \times \mathbb{P}^{n_m}$$

where

$$\mathbb{P}^n: \quad \begin{cases} p_i(Z^0, \dots, Z^n) & = P_{i_1 \dots i_i} Z^{i_1} \dots Z^{i_i} = 0 \\ p_i(\lambda Z^0, \dots, \lambda Z^n) & = \lambda^i p_i(Z^0, \dots, Z^n) \end{cases}$$

[Green, Hübsch (1987); Hübsch (1992)]

Representation of the Output

CICY can be generalised to m projective spaces and k equations. The problem is thus mapped to:

$$\mathcal{R}: \quad \mathbb{N}^{m \times k} \quad \longrightarrow \quad \mathbb{N}$$

$$\left[\begin{array}{c|ccc} \mathbb{P}^{n_1} & a_1^1 & \dots & a_k^1 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{P}^{n_m} & a_1^m & \dots & a_k^m \end{array} \right] \longrightarrow h^{1,1} \quad \text{or} \quad h^{2,1}$$

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Machine Learning Approach

What is $\mathcal{R}$ in **machine learning** approach?

$$\mathcal{R}(M) \rightarrow \mathcal{R}_n(M; w) \rightarrow \hat{h}^{p,q} \quad \text{s.t.} \quad \exists n > M > 0 \quad | \quad \mathcal{L}_n(\hat{h}^{p,q}, h^{p,q}) < \epsilon \quad \forall \epsilon > 0$$

Machine Learning

- **optimisation problem** $\Rightarrow$ gradient descent (or similar)

Machine Learning

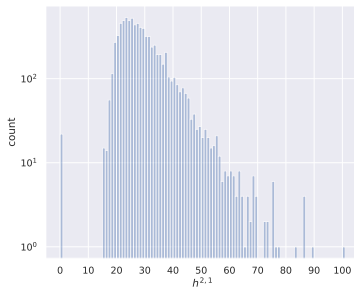
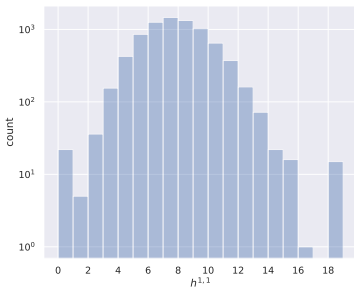
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Machine Learning

- **optimisation problem** $\Rightarrow$ **gradient descent** (or similar)
- use **various algorithms** and exploit **large datasets** (more training)
- intersection of **computer science, mathematics and physics**
- provide in-depth **data analysis** of the datasets



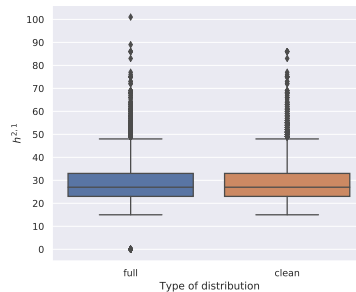
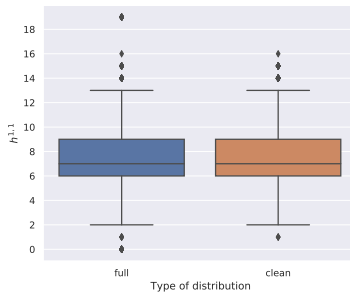
[Green *et al.* (1987)]

Dataset

- 7890 CICY manifolds (full dataset)
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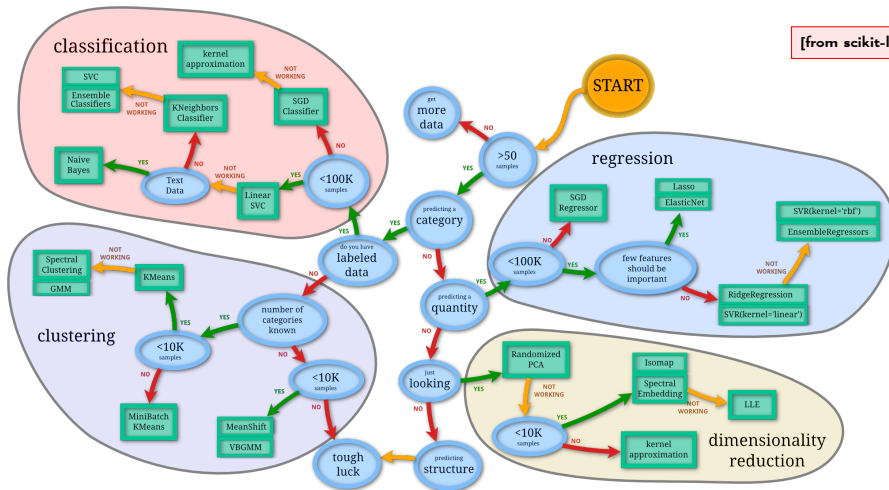


Exploratory Data Analysis

exploratory data analysis → feature **selection** → Hodge numbers

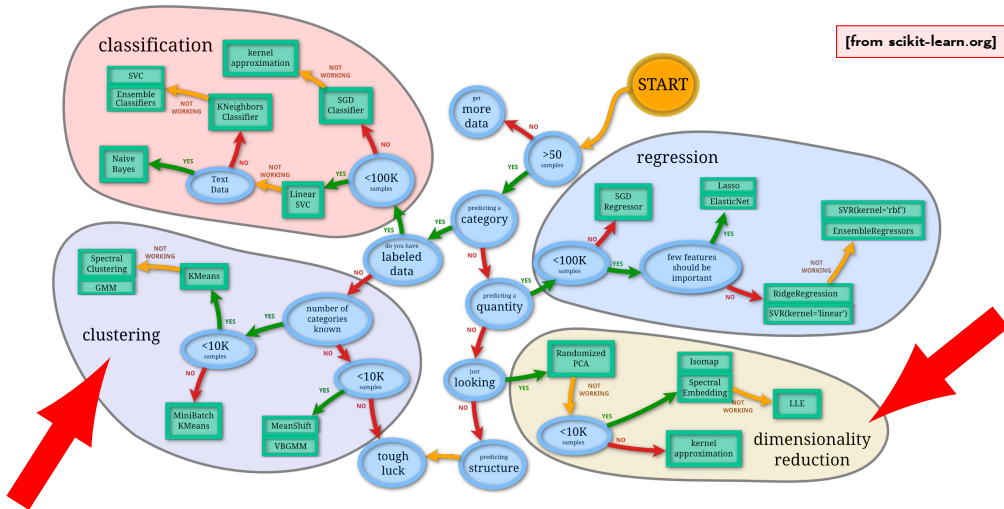
[Ruehle (2020); Erbin, RF (2020)]

[from scikit-learn.org]



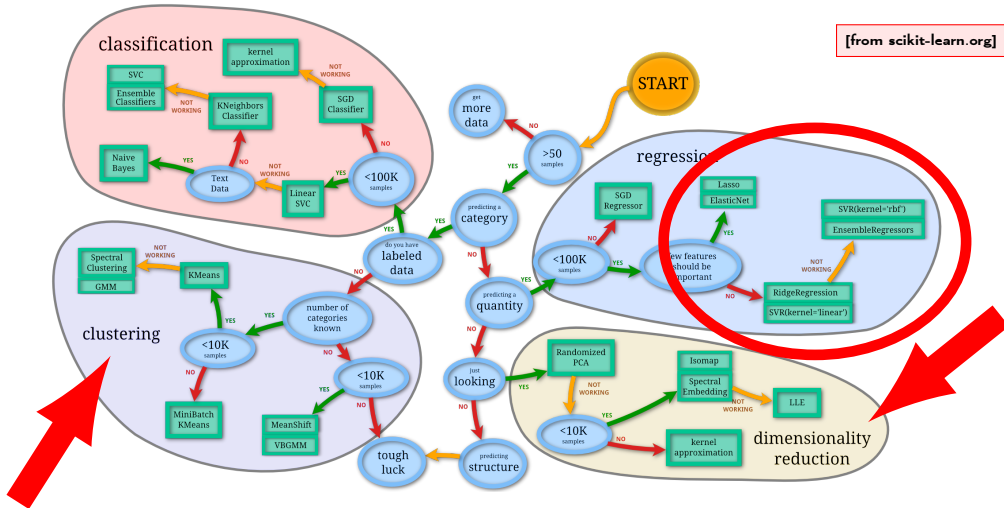
Machine Learning

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A Word on PCA

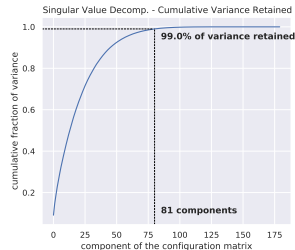
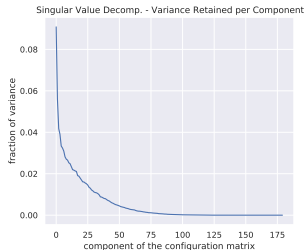
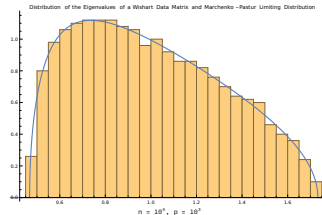
What is PCA for a $X \in \mathbb{R}^{n \times p}$?

- project data such that **variance is maximised**
- **eigenvectors** of XX^T or the **singular values** of X
- isolate **signal** from **background**
- ease the ML job of finding a better representation of the input

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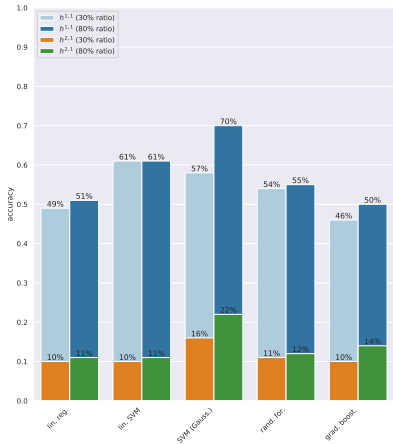
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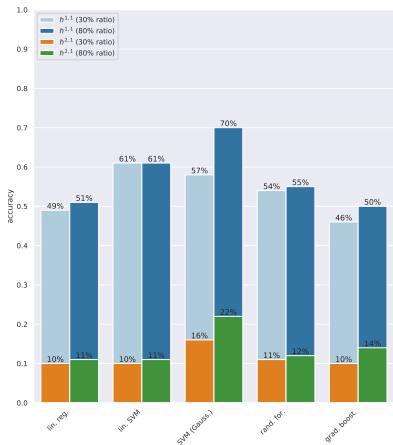
Machine Learning Results

Configuration Matrix Only

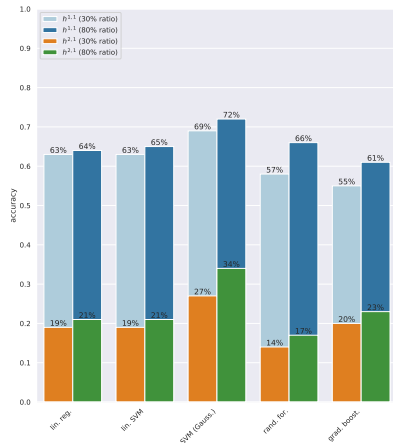


Machine Learning Results

Configuration Matrix Only



Best Training Set [Erbin, RF (2020)]



Artificial Intelligence and Neural Networks

- use **gradient descent** to optimise **weights**
- learn highly **non linear** representations of the input
- can be “large” to have enough parameters
- can be “deep” to learn **complicated functions**

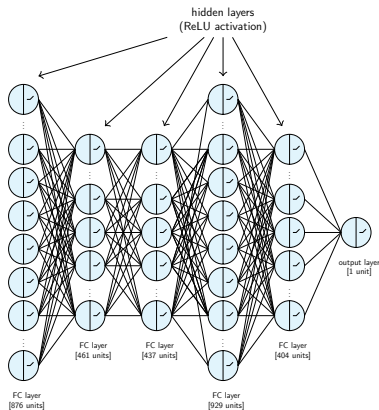
Layers

fully connected: $\phi(a^{(i)} \{l\} \cdot W^{(l)} + b^{(l)} \mathbf{1})$

convolutional: $\phi(a^{(i)} \{l\} * W^{(l)} + b^{(l)} \mathbf{1})$

Non linearity ensured by:

$$\phi(z) = \text{ReLU}(z) = \max(0, z)$$

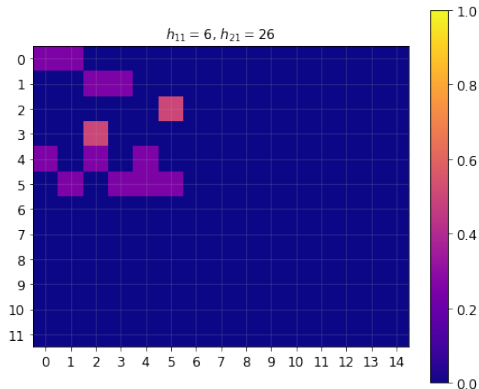


[rendition of the neural network in Bull et al. (2018)]

Convolutional Neural Networks

Why convolutional?

- retain **spacial awareness**
- smaller **no. of parameters**
($\approx 2 \times 10^5$ vs. $\approx 2 \times 10^6$)
- weights are **shared**
- CNNs isolate “**defining features**”
- find patterns as in **computer vision**



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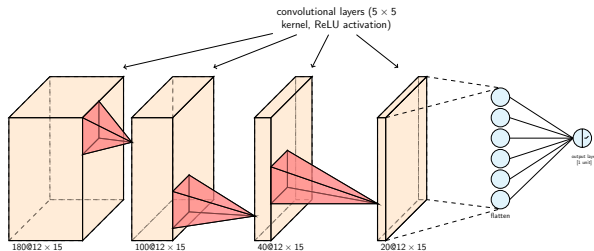
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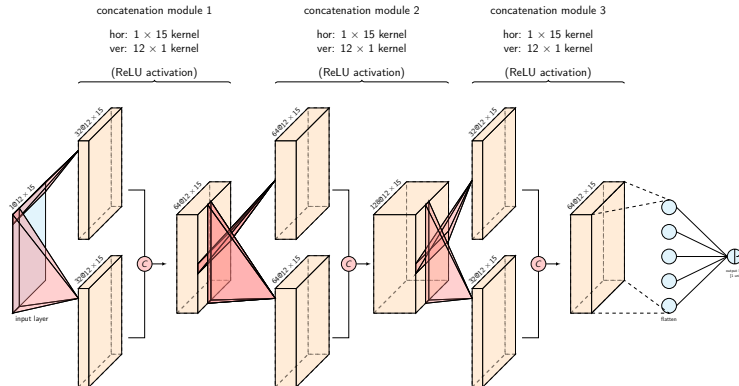
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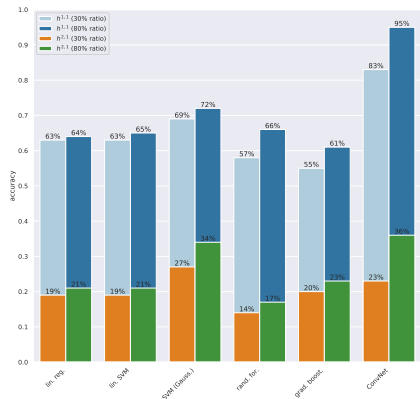
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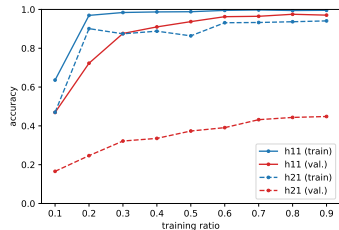
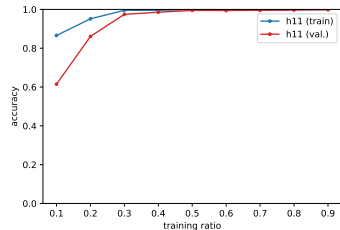
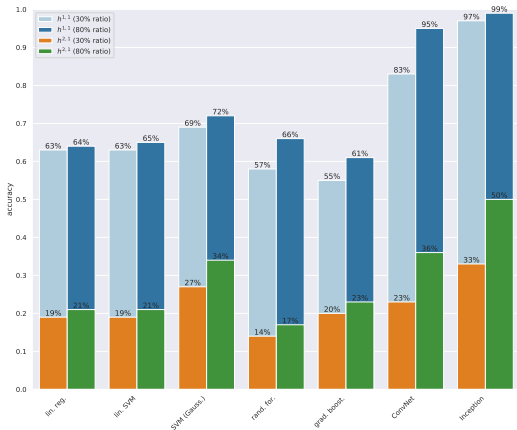
Deep Learning Topology with Computer Vision

Best Training Set [Erbin, RF (2020)]



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Best Training Set [Erbin, RF (2020)]



[see Erbin's talk at *string_data 2020*]

A Few Comments and Future Directions

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- interdisciplinary approach = win-win situation!

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What now?

- representation learning $\Rightarrow$ what is the best way to represent CICYs?
- study invariances $\Rightarrow$ invariances should not influence the result (graph representations?)
- higher dimensions $\Rightarrow$ what about CICY 4-folds?
- geometric deep learning $\Rightarrow$ explain the geometry of the “AI” behind deep learning!
- reinforcement learning $\Rightarrow$ give the rules, not the result!

The End?

- D-branes at angles and defect CFT → spin and twist fields
- time dependent orbifolds → strings and divergences
- deep learning → CICY and topological properties



THANK YOU