D-branes and Deep Learning

Theoretical and Computational Aspects in String Theory

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Action Principle and Conformal Symmetry

Polyakov's Action

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Symmetries:

- **Poincaré transf.** $X'^\mu = \Lambda^\mu{}_\nu X^\nu + c^\mu$
- **2D diff.** $\gamma'_{\alpha\beta} = \left(J^{-1}\right)_{\alpha\beta}{}^{\lambda\rho} \gamma_{\lambda\rho}$
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Conformal symmetry:

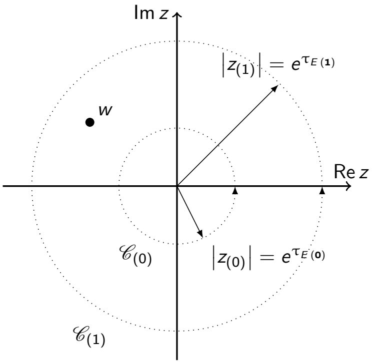
- **vanishing** stress-energy tensor: $\mathcal{T}_{\alpha\beta} = 0$
- **traceless** stress-energy tensor: $\mathrm{tr}\, \mathcal{T} = 0$
- **conformal gauge** $\gamma_{\alpha\beta} = e^{\phi} \eta_{\alpha\beta}$

Action Principle and Conformal Symmetry

Let $z = e^{\tau_E + i\sigma} \Rightarrow \bar{\partial}\mathcal{T}(z) = \partial\overline{\mathcal{T}}(\bar{z}) = 0$:

$$\mathcal{T}(z)\,\Phi_h(w) \overset{z\to w}{\sim} \frac{h}{(z-w)^2}\Phi_h(w) + \frac{1}{z-w}\partial_w\Phi_h(w)$$

$$\mathcal{T}(z)\,\mathcal{T}(w) \overset{z\to w}{\sim} \frac{\frac{c}{2}}{(z-w)^4} + \mathcal{O}\big((z-w)^{-2}\big)$$



Virasoro algebra $\mathscr{V} \oplus \overline{\mathscr{V}}$

$$\begin{aligned} \left[\overset{(-)}{L}_n, \overset{(-)}{L}_m\right] &= (n-m)\overset{(-)}{L}_{n+m} + \frac{c}{12}n\left(n^2-1\right)\delta_{n+m,0} \\ \left[L_n, \overline{L}_m\right] &= 0 \end{aligned}$$

Action Principle and Conformal Symmetry

Superstrings in D dimensions:

$$\mathcal{T}(z) = -\frac{1}{\alpha'}\partial X(z)\cdot\partial X(z) - \frac{1}{2}\psi(z)\cdot\partial\psi(z) \quad \Rightarrow \quad c = \frac{3}{2}D$$

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$(\lambda, 0)$ / $(1-\lambda, 0)$ Ghost System

Introduce anti-commuting $(b,\, c)$ and commuting $(\beta,\, \gamma)$ conformal fields:

$$S_{\text{ghost}}[b,\, c,\, \beta,\, \gamma] = \frac{1}{2\pi}\iint \mathrm{d}z\, \mathrm{d}\bar{z}\, \left(b(z)\,\bar{\partial}c(z) + \beta(z)\,\bar{\partial}\gamma(z)\right)$$

where $\lambda_b = 2$ and $\lambda_c = -1$, and $\lambda_\beta = \frac{3}{2}$ and $\lambda_\gamma = -\frac{1}{2}$.

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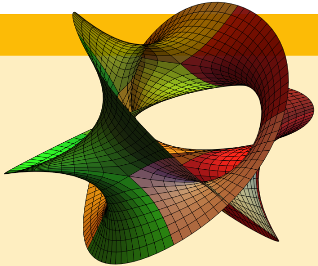
Consequence:

$$c_{\text{full}} = c + c_{\text{ghost}} = 0 \quad\Leftrightarrow\quad D = 10.$$

Extra Dimensions and Compactification

Compactification

$$\mathscr{M}^{1,9} = \mathscr{M}^{1,3} \otimes \mathscr{X}_6$$



- $\mathscr{X}_6$ is a **compact** manifold
- $N = 1$ **supersymmetry** is preserved in 4D
- algebra of $\mathrm{SU}(3) \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1)$ in arising **gauge group**

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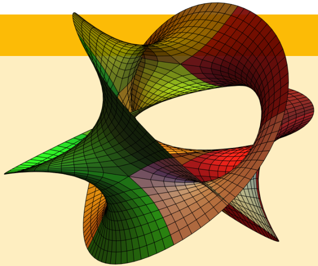

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Kähler manifolds (M, g) such that

- $\dim_{\mathbb{C}} M = m$
- $\mathrm{Hol}(g) \subseteq \mathrm{SU}(m)$
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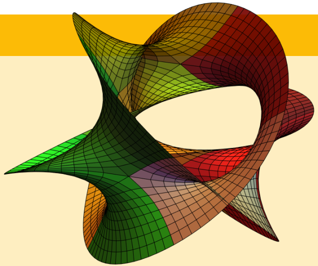

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Characterised by Hodge numbers

$$h^{r,s} = \dim_{\mathbb{C}} H^{r,s}_{\bar{\partial}}(M, \mathbb{C})$$

counting the no. of harmonic (r, s)-forms.

D-branes and Open Strings

Polyakov's action naturally introduces Neumann b.c.:

$$\partial_\sigma X(\tau,\sigma)\Big|_{\sigma=0}^{\sigma=\ell} = 0$$

satisfied by **open and closed** strings living in D dimensions s.t. $\square X = 0$.

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T-duality

$$X(z, \bar{z}) = X(z) + \overline{X}(\bar{z}) \quad \overset{T}{\Rightarrow} \quad X(z) - \overline{X}(\bar{z}) = Y(z, \bar{z}) = Y(z) + \overline{Y}(\bar{z})$$

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Resulting effect (repeated $p \leq D-1$ times) leads to Dirichlet b.c.:

$$\partial_\sigma X^i(\tau,\sigma)\Big|_{\sigma=0}^{\sigma=\ell} = 0 \quad \overset{T}{\Rightarrow} \quad \partial_\tau Y^i(\tau,\sigma)\Big|_{\sigma=0}^{\sigma=\ell} = 0 \quad \forall i = 1, 2, \dots, p$$

thus **open strings** can be **constrained** to $D(D-p-1)$-branes.

D-branes and Open Strings

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Massless Spectrum (*irrep* of little group $\mathrm{SO}(D-2)$)

$$\mathcal{A}^{\mu} \quad \leftrightarrow \quad \alpha^{\mu}_{-1} \left|0\right\rangle \qquad \longrightarrow \qquad \begin{array}{lcll} \mathcal{A}^{A} & \leftrightarrow & \alpha^{A}_{-1} \left|0\right\rangle, & A = 0,\, 1,\, \dots,\, p \\ \mathcal{A}^{a} & \leftrightarrow & \alpha^{a}_{-1} \left|0\right\rangle, & a = 1,\, 2,\, \dots,\, D-p-1 \end{array}$$

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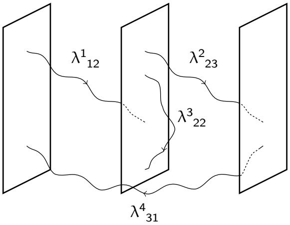

Introducing **Chan–Paton factors** $\lambda^r{}_{ij}$, when branes are **coincident**:

$$\bigoplus_{r=1}^{N} \mathrm{U}_r(1) \quad \longrightarrow \quad \mathrm{U}(N)$$

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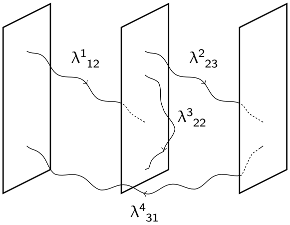

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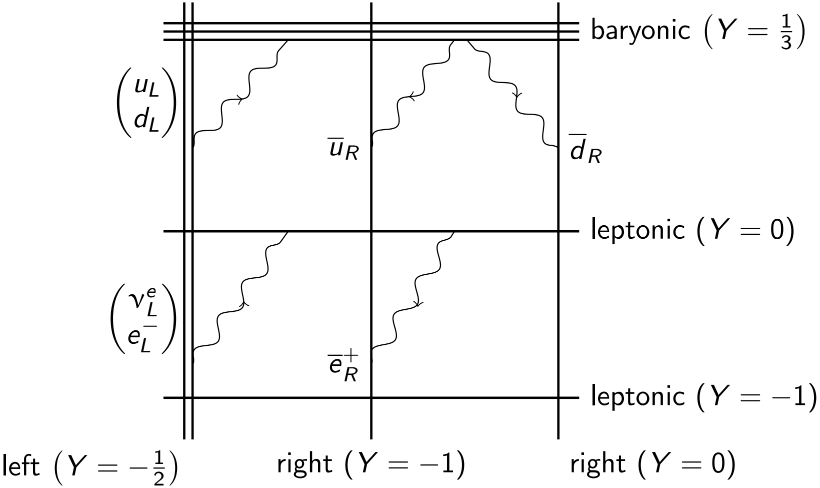
$\boxed{\text{Build gauge bosons, fermions and scalars.}}$

Standard Model-like Scenarios



Intersecting D-branes

Consider N intersecting $D6$-branes filling $\mathscr{M}^{1,3}$ and **embedded in** $\mathbb{R}^6$

Twist Fields Correlators

$$\left\langle \prod_{t=1}^{N_B} \sigma_{\mathrm{M}_{(t)}}\left(x_{(t)}\right) \right\rangle = \mathcal{N}\left(\left\{ x_{(t)},\, \mathrm{M}_{(t)} \right\}_{1 \le t \le N_B} \right) e^{-S_{E\,(\mathbf{cl})}\left(\left\{ x_{(t)},\, \mathrm{M}_{(t)} \right\}_{1 \le t \le N_B} \right)}$$

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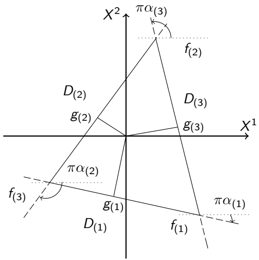

D-branes in **factorised** internal space:

- **embedded as lines** in $\mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2$
- **relative rotations** are $\mathrm{SO}(2) \simeq \mathrm{U}(1)$ elements
- $S_{E\,(\mathrm{cl})}\left(\left\{ x_{(t)},\, \mathrm{M}_{(t)} \right\}_{1 \le t \le N_B} \right) \sim \mathrm{Area}\left(\left\{ f_{(t)},\, \mathrm{R}_{(t)} \right\}_{1 \le t \le N_B} \right)$

$\mathrm{SO}(4)$ Rotations

Consider $\boxed{\mathbb{R}^4 \times \mathbb{R}^2}$ (focus on $\mathbb{R}^4$):

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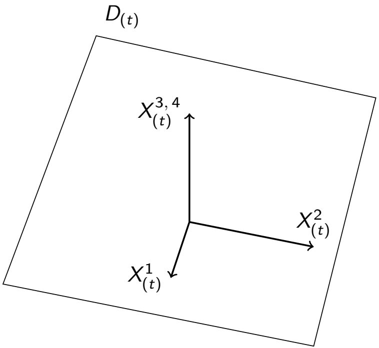

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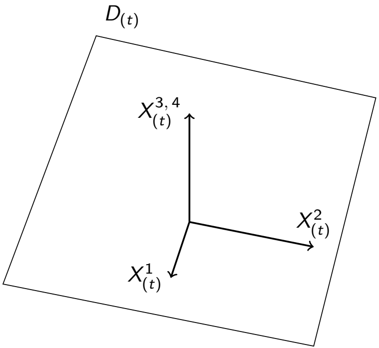

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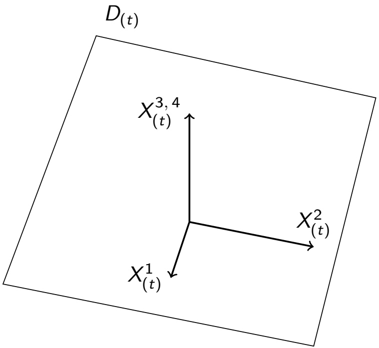

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that is

$$\left[R_{(t)}\right] = \left\{R_{(t)} \sim \mathcal{O}_{(t)} R_{(t)}\right\}$$

Boundary Conditions

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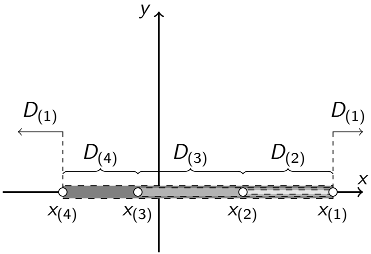
- consider $u = x + iy = e^{\tau_e + i\sigma}$ and $\overline{u} = u^*$
- let $x_{(t)} < x_{(t-1)}$ be the **worldsheet intersection points** on **real axis**
- $X^{1,2}_{(t)}$ are **Neumann**, $X^{3,4}_{(t)}$ are **Dirichlet**



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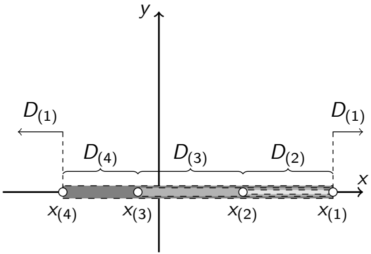

Branch Cuts and Discontinuities for $x \in D_{(t)}$

$$\begin{cases} \partial_u X(x + i0^+) & = U_{(t)} \cdot \partial_{\overline{u}} \overline{X}(x - i0^+) = \left[R^{-1}_{(t)} \cdot (\sigma_3 \otimes \mathbb{1}_2) \cdot R_{(t)} \right] \cdot \partial_{\overline{u}} \overline{X}(x - i0^+) \\ X(x_{(t)}, x_{(t)}) & = f_{(t)} \end{cases}$$

Doubling Trick and Spinor Representation

Doubling Trick

$$\partial_z \mathcal{X}(z) = \begin{cases} \partial_u X(u) & \text{if } z \in \mathscr{H}_{>}^{(\bar{t})} \\ U_{(\bar{t})}\, \partial_{\bar{u}} \overline{X}(\bar{u}) & \text{if } z \in \mathscr{H}_{<}^{(\bar{t})} \end{cases} \quad \Rightarrow \quad \begin{aligned} \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i}\delta_+) &= \mathcal{U}_{(t,\, t+1)}\, \partial_z \mathcal{X}(x_{(t)} + \delta_+), \\ \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i}\delta_-) &= \widetilde{\mathcal{U}}_{(t,\, t+1)}\, \partial_z \mathcal{X}(x_{(t)} + \delta_-), \end{aligned}$$

where $\mathscr{H}_{\gtrless}^{(t)} = \left\{ z \in \mathbb{C} \mid \operatorname{Im} z \gtrless 0 \text{ or } z \in D_{(t)} \right\}$ and $\delta_\pm = \eta \pm i0^+$.

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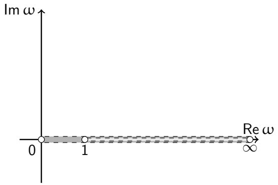
Use Pauli matrices $\tau = (i\,\mathbb{1}_2, \vec{\sigma})$:

$$\partial_z \mathcal{X}_{(s)}(z) = \partial_z \mathcal{X}^I(z)\, \tau_I \quad \Rightarrow \quad \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i}\,\delta_\pm) = \overset{(\sim)}{\mathcal{L}}_{(t,\,t+1)}\, \partial_z \mathcal{X}(x_{(t)} + \delta_\pm)\, \overset{(\sim)}{\mathcal{R}}_{(t,\,t+1)}$$

where

$$\overset{(\sim)}{\mathcal{L}}_{(t,\,t+1)} \in \mathrm{SU}(2)_L \quad \text{and} \quad \overset{(\sim)}{\mathcal{R}}_{(t,\,t+1)} \in \mathrm{SU}(2)_R$$

Hypergeometric Basis



Sum over all contributions:

$$\partial_z \mathcal{X}(z) = \sum_{l,\, r} c_{lr} \, (-\omega_z)^{A_{lr}} \, (1-\omega_z)^{B_{lr}} \, B^{(L)}_{0,\, l}(\omega_z) \left(B^{(R)}_{0,\, r}(\omega_z) \right)^T$$

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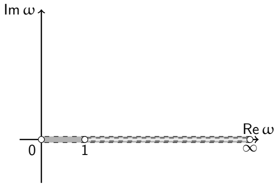

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Basis of Solutions

$$B_{0,\, n}(\omega_z) = \begin{pmatrix} 1 & 0 \\ 0 & K_n \end{pmatrix} \begin{pmatrix} \frac{1}{\Gamma(c_n)}\, {}_2F_1\left(a_n,\, b_n;\, c_n;\, \omega_z\right) \\ \left(-\omega_z\right)^{1-c_n} \frac{1}{\Gamma(2-c_n)}\, {}_2F_1\left(a_n+1-c_n,\, b_n+1-c_n;\, 2-c_n;\, \omega_z\right) \end{pmatrix}$$

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Operations sequence:

1. rotation matrix $=$ monodromy matrix

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3. finite action $\Rightarrow$ 2 solutions (no. of d.o.f. is correctly saturated)

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1. rotation matrix = monodromy matrix
2. contiguity relations $\Rightarrow$ independent hypergeometrics
3. finite action $\Rightarrow$ 2 solutions (no. of d.o.f. is correctly saturated)
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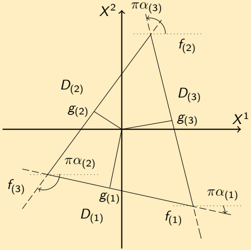
Physical Interpretation



$$
\begin{aligned}
S_{\mathbb{R}^4}\Big|_{\text{on-shell}} &= \frac{1}{2\pi\alpha'}\sum_{t=1}^{3}\left(\frac{1}{2}\left|g_{(t)}^{\perp}\right|\left|f_{(t-1)} - f_{(t)}\right|\right) \\
&= \text{Area}\left(\left\{f_{(t)}\right\}\right)
\end{aligned}
$$

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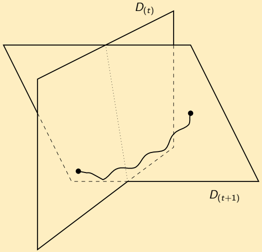
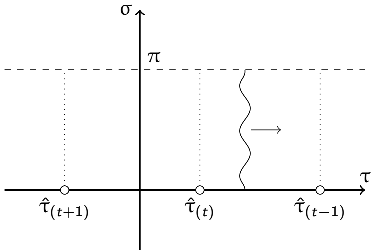


Fermions on the Strip



Action of Boundary Changing Operators

$$\begin{cases} \psi^i_-(\tau,0) & = \left(R_{(t)}\right)^I{}_J\,\psi^J_+(\tau,0) \quad \text{for } \tau \in \left(\hat{\tau}_{(t)},\, \hat{\tau}_{(t-1)}\right) \\ \psi^I_-(\tau,\pi) & = -\psi^I_+(\tau,\pi) \quad \text{for } \tau \in \mathbb{R} \end{cases}$$

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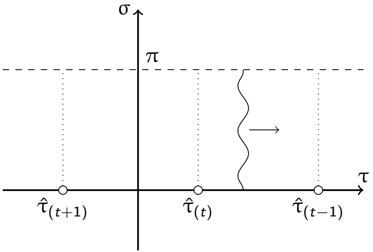

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Stress-energy Tensor

$$\mathcal{T}_{\pm\pm}(\xi_\pm) = -i\,\frac{T}{4}\,\psi^*_{\pm,I}(\xi_\pm)\,\overleftrightarrow{\partial}\psi^I_\pm(\xi_\pm) \quad \Rightarrow \quad \begin{cases} \dot{\mathrm{H}}(\tau) &= 0 \Leftrightarrow \tau \in \left(\tau_{(t)},\, \tau_{(t-1)}\right) \\ \dot{\mathrm{P}}(\tau) &\neq 0 \end{cases}$$

BBB

b

CCC

C