

D-branes and Deep Learning

Theoretical and Computational Aspects in String Theory

Riccardo Finotello

Scuola di Dottorato in Fisica e Astrofisica

Università degli Studi di Torino
and

I.N.F.N. – sezione di Torino

15th December 2020



Contents

Conformal Symmetry and Geometry of the Worldsheet

Preliminary Concepts and Tools

D-branes Intersecting at Angles

Fermions and Point-like Defect CFT

Cosmological Backgrounds and Divergences

Orbifolds and Cosmological Toy Models

Null Boost Orbifold

Deep Learning the Geometry of String Theory

Machine Learning and Deep Learning

Machine Learning for String Theory

AI Implementations for Geometry and Strings

Contents

Conformal Symmetry and Geometry of the Worldsheet

Preliminary Concepts and Tools

D-branes Intersecting at Angles

Fermions and Point-like Defect CFT

Cosmological Backgrounds and Divergences

Orbifolds and Cosmological Toy Models

Null Boost Orbifold

Deep Learning the Geometry of String Theory

Machine Learning and Deep Learning

Machine Learning for String Theory

AI Implementations for Geometry and Strings

Action Principle and Conformal Symmetry

Polyakov's Action

$$S_P[\gamma, X, \psi] = -\frac{1}{4\pi} \int_{-\infty}^{+\infty} d\tau \int_0^\ell d\sigma \sqrt{-\det \gamma} \gamma^{\alpha\beta} \left(\frac{2}{\alpha'} \partial_\alpha X^\mu \partial_\beta X^\nu + \psi^\mu \rho_\alpha \partial_\beta \psi^\nu \right) \eta_{\mu\nu}$$

Action Principle and Conformal Symmetry

Polyakov's Action

$$S_P[\gamma, X, \psi] = -\frac{1}{4\pi} \int_{-\infty}^{+\infty} d\tau \int_0^\ell d\sigma \sqrt{-\det \gamma} \gamma^{\alpha\beta} \left(\frac{2}{\alpha'} \partial_\alpha X^\mu \partial_\beta X^\nu + \psi^\mu \rho_\alpha \partial_\beta \psi^\nu \right) \eta_{\mu\nu}$$

Symmetries:

- **Poincaré transf.** $X'^\mu = \Lambda^\mu_\nu X^\nu + c^\mu$
- **2D diff.** $\gamma'_{\alpha\beta} = (J^{-1})_{\alpha\beta}^{\lambda\rho} \gamma_{\lambda\rho}$
- **Weyl transf.** $\gamma'_{\alpha\beta} = e^{2\omega} \gamma_{\alpha\beta}$

Action Principle and Conformal Symmetry

Polyakov's Action

$$S_P[\gamma, X, \psi] = -\frac{1}{4\pi} \int_{-\infty}^{+\infty} d\tau \int_0^\ell d\sigma \sqrt{-\det \gamma} \gamma^{\alpha\beta} \left(\frac{2}{\alpha'} \partial_\alpha X^\mu \partial_\beta X^\nu + \psi^\mu \rho_\alpha \partial_\beta \psi^\nu \right) \eta_{\mu\nu}$$

Symmetries:

- **Poincaré transf.** $X'^\mu = \Lambda^\mu_\nu X^\nu + c^\mu$
- **2D diff.** $\gamma'_{\alpha\beta} = (J^{-1})_{\alpha\beta}^{\lambda\rho} \gamma_{\lambda\rho}$
- **Weyl transf.** $\gamma'_{\alpha\beta} = e^{2\omega} \gamma_{\alpha\beta}$

Conformal symmetry:

- **vanishing** stress-energy tensor: $\mathcal{T}_{\alpha\beta} = 0$
- **traceless** stress-energy tensor: $\text{tr } \mathcal{T} = 0$
- **conformal gauge** $\gamma_{\alpha\beta} = e^\Phi \eta_{\alpha\beta}$

Action Principle and Conformal Symmetry

Superstrings in D dimensions:

$$\mathcal{T}(z) = -\frac{1}{\alpha'} \partial X(z) \cdot \partial X(z) - \frac{1}{2} \psi(z) \cdot \partial \psi(z) \quad \Rightarrow \quad c = \frac{3}{2}D$$

Action Principle and Conformal Symmetry

Superstrings in D dimensions:

$$\mathcal{T}(z) = -\frac{1}{\alpha'} \partial X(z) \cdot \partial X(z) - \frac{1}{2} \psi(z) \cdot \partial \psi(z) \quad \Rightarrow \quad c = \frac{3}{2} D$$

$(\lambda, 0) / (1 - \lambda, 0)$ **Ghost System**

Introduce anti-commuting (b, c) and commuting (β, γ) conformal fields:

$$S_{\text{ghost}}[b, c, \beta, \gamma] = \frac{1}{2\pi} \iint dz d\bar{z} (b(z) \bar{\partial} c(z) + \beta(z) \bar{\partial} \gamma(z))$$

where $\lambda_b = 2$ and $\lambda_c = -1$, and $\lambda_\beta = \frac{3}{2}$ and $\lambda_\gamma = -\frac{1}{2}$.

[Friedan, Martinec, Shenker (1986)]

Action Principle and Conformal Symmetry

Superstrings in D dimensions:

$$\mathcal{T}(z) = -\frac{1}{\alpha'} \partial X(z) \cdot \partial X(z) - \frac{1}{2} \psi(z) \cdot \partial \psi(z) \quad \Rightarrow \quad c = \frac{3}{2}D$$

$(\lambda, 0) / (1 - \lambda, 0)$ Ghost System

Introduce anti-commuting (b, c) and commuting (β, γ) conformal fields:

$$S_{\text{ghost}}[b, c, \beta, \gamma] = \frac{1}{2\pi} \iint dz d\bar{z} (b(z) \bar{\partial} c(z) + \beta(z) \bar{\partial} \gamma(z))$$

where $\lambda_b = 2$ and $\lambda_c = -1$, and $\lambda_\beta = \frac{3}{2}$ and $\lambda_\gamma = -\frac{1}{2}$.

[Friedan, Martinec, Shenker (1986)]

Consequence:

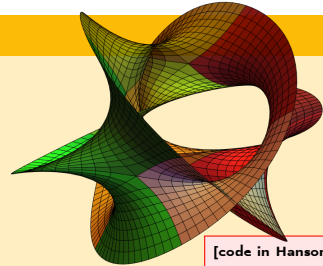
$$c_{\text{full}} = c + c_{\text{ghost}} = 0 \quad \Leftrightarrow \quad D = 10.$$

Extra Dimensions and Compactification

Compactification

$$\mathcal{M}^{1,9} = \mathcal{M}^{1,3} \otimes \mathcal{X}_6$$

- $\mathcal{X}_6$ is a **compact** manifold
- $N = 1$ **supersymmetry** is preserved in 4D
- algebra of $SU(3) \otimes SU(2) \otimes U(1)$ in arising **gauge group**



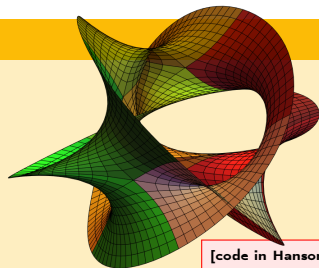
[code in Hanson (1994)]

Extra Dimensions and Compactification

Compactification

$$\mathcal{M}^{1,9} = \mathcal{M}^{1,3} \otimes \mathcal{X}_6$$

- $\mathcal{X}_6$ is a **compact** manifold
- $N = 1$ **supersymmetry** is preserved in 4D
- algebra of $SU(3) \otimes SU(2) \otimes U(1)$ in arising **gauge group**



[code in Hanson (1994)]

Kähler manifolds (M, g) such that

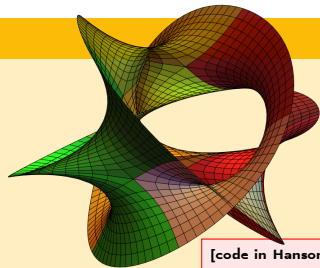
- $\dim_{\mathbb{C}} M = m$
- $\text{Hol}(g) \subseteq SU(m)$
- g is Ricci-flat (equiv. $c_1(M)$ vanishes)

Extra Dimensions and Compactification

Compactification

$$\mathcal{M}^{1,9} = \mathcal{M}^{1,3} \otimes \mathcal{X}_6$$

- $\mathcal{X}_6$ is a **compact** manifold
- $N = 1$ **supersymmetry** is preserved in 4D
- algebra of $SU(3) \otimes SU(2) \otimes U(1)$ in arising **gauge group**



[code in Hanson (1994)]

Kähler manifolds (M, g) such that

- $\dim_{\mathbb{C}} M = m$
- $\text{Hol}(g) \subseteq SU(m)$
- g is Ricci-flat (equiv. $c_1(M)$ vanishes)

Characterised by Hodge numbers

$$h^{r,s} = \dim_{\mathbb{C}} H_{\bar{\partial}}^{r,s}(M, \mathbb{C})$$

counting the no. of harmonic (r, s) -forms.

D-branes and Open Strings

Polyakov's action naturally introduces Neumann b.c.:

$$\partial_\sigma X(\tau, \sigma) \Big|_{\sigma=0}^{\sigma=\ell} = 0$$

satisfied by **open and closed** strings living in D dimensions s.t. $\square X = 0$.

D-branes and Open Strings

Polyakov's action naturally introduces **Neumann b.c.:**

$$\partial_\sigma X(\tau, \sigma) \Big|_{\sigma=0}^{\sigma=\ell} = 0$$

satisfied by **open and closed** strings living in D dimensions s.t. $\square X = 0$.

T-duality

$$X(z, \bar{z}) = X(z) + \bar{X}(\bar{z}) \quad \xrightarrow{T} \quad X(z) - \bar{X}(\bar{z}) = Y(z, \bar{z}) = Y(z) + \bar{Y}(\bar{z})$$

D-branes and Open Strings

Polyakov's action naturally introduces Neumann b.c.:

$$\partial_\sigma X(\tau, \sigma) \Big|_{\sigma=0}^{\sigma=\ell} = 0$$

satisfied by **open and closed** strings living in D dimensions s.t. $\square X = 0$.

T-duality

$$X(z, \bar{z}) = X(z) + \bar{X}(\bar{z}) \xrightarrow{T} X(z) - \bar{X}(\bar{z}) = Y(z, \bar{z}) = Y(z) + \bar{Y}(\bar{z})$$

Resulting effect (repeated $p \leq D - 1$ times) leads to Dirichlet b.c.:

$$\partial_\sigma X^i(\tau, \sigma) \Big|_{\sigma=0}^{\sigma=\ell} = 0 \xrightarrow{T} \partial_\tau Y^i(\tau, \sigma) \Big|_{\sigma=0}^{\sigma=\ell} = 0 \quad \forall i = 1, 2, \dots, p$$

thus **open strings** can be **constrained** to $D(D - p - 1)$ -branes.

[Polchinski (1995, 1996)]

D-branes and Open Strings

Introducing Dp -branes breaks $\text{ISO}(1, D - 1) \rightarrow \text{ISO}(1, p) \otimes \text{SO}(D - 1 - p)$.

D-branes and Open Strings

Introducing Dp -branes breaks $\text{ISO}(1, D-1) \rightarrow \text{ISO}(1, p) \otimes \text{SO}(D-1-p)$.

Massless Spectrum (*irrep* of little group $\text{SO}(D-2)$)

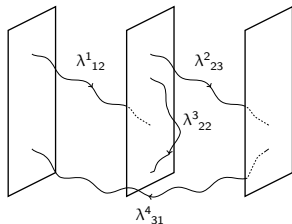
$$\mathcal{A}^\mu \leftrightarrow \alpha_{-1}^\mu |0\rangle \longrightarrow \begin{array}{ll} \mathcal{A}^A \leftrightarrow \alpha_{-1}^A |0\rangle, & A = 0, 1, \dots, p \\ \mathcal{A}^a \leftrightarrow \alpha_{-1}^a |0\rangle, & a = 1, 2, \dots, D-p-1 \end{array}$$

D-branes and Open Strings

Introducing Dp -branes breaks $\text{ISO}(1, D-1) \rightarrow \text{ISO}(1, p) \otimes \text{SO}(D-1-p)$.

Massless Spectrum (*irrep* of little group $\text{SO}(D-2)$)

$$\mathcal{A}^\mu \leftrightarrow \alpha_{-1}^\mu |0\rangle \quad \longrightarrow \quad \begin{aligned} \mathcal{A}^A &\leftrightarrow \alpha_{-1}^A |0\rangle, & A = 0, 1, \dots, p \\ \mathcal{A}^a &\leftrightarrow \alpha_{-1}^a |0\rangle, & a = 1, 2, \dots, D-p-1 \end{aligned}$$



Introducing **Chan–Paton factors** λ^r_{ij} , when branes are **coincident**:

[Chan, Paton (1969)]

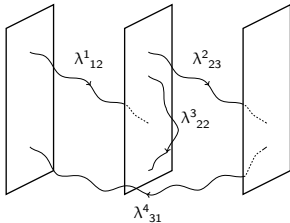
$$\bigoplus_{r=1}^N \text{U}_r(1) \quad \longrightarrow \quad \text{U}(N)$$

D-branes and Open Strings

Introducing Dp -branes breaks $\text{ISO}(1, D-1) \rightarrow \text{ISO}(1, p) \otimes \text{SO}(D-1-p)$.

Massless Spectrum (*irrep* of little group $\text{SO}(D-2)$)

$$\mathcal{A}^\mu \leftrightarrow \alpha_{-1}^\mu |0\rangle \quad \longrightarrow \quad \begin{aligned} \mathcal{A}^A &\leftrightarrow \alpha_{-1}^A |0\rangle, & A = 0, 1, \dots, p \\ \mathcal{A}^a &\leftrightarrow \alpha_{-1}^a |0\rangle, & a = 1, 2, \dots, D-p-1 \end{aligned}$$



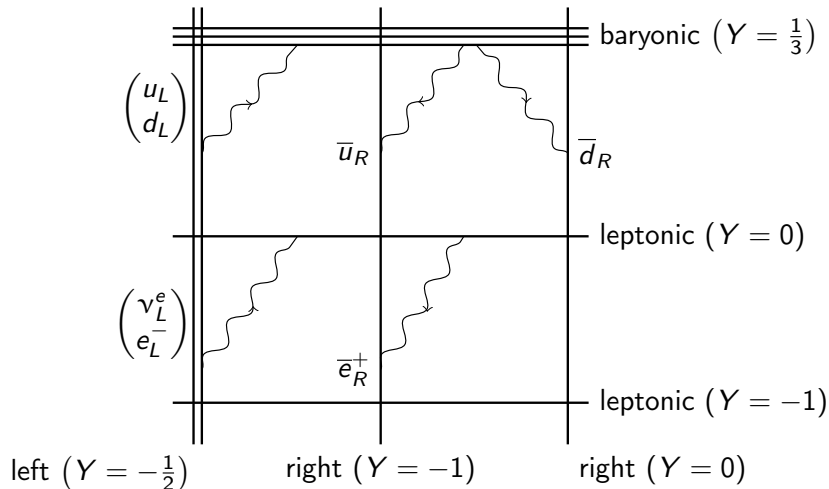
Introducing **Chan–Paton factors** λ^r_{ij} , when branes are **coincident**:

[Chan, Paton (1969)]

$$\bigoplus_{r=1}^N \text{U}_r(1) \quad \longrightarrow \quad \text{U}(N)$$

Build gauge bosons, fermions and scalars.

Standard Model-like Scenarios



[Zwiebach (2009)]

Intersecting D-branes

Consider N intersecting $D6$ -branes filling $\mathcal{M}^{1,3}$ and embedded in $\mathbb{R}^6$

Twist Fields Correlators

$$\left\langle \prod_{t=1}^{N_B} \sigma_{M_{(t)}}(x_{(t)}) \right\rangle = \mathcal{N} \left(\{x_{(t)}, M_{(t)}\}_{1 \leq t \leq N_B} \right) e^{-S_{E(\text{cl})} \left(\{x_{(t)}, M_{(t)}\}_{1 \leq t \leq N_B} \right)}$$

Intersecting D-branes

Consider N intersecting $D6$ -branes filling $\mathcal{M}^{1,3}$ and embedded in $\mathbb{R}^6$

Twist Fields Correlators

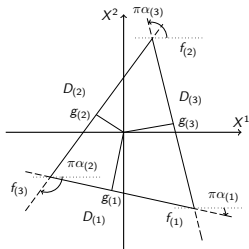
$$\left\langle \prod_{t=1}^{N_B} \sigma_{M_{(t)}}(x_{(t)}) \right\rangle = \mathcal{N} \left(\{x_{(t)}, M_{(t)}\}_{1 \leq t \leq N_B} \right) e^{-S_E(\text{cl}) \left(\{x_{(t)}, M_{(t)}\}_{1 \leq t \leq N_B} \right)}$$

Intersecting D-branes

Consider N intersecting $D6$ -branes filling $\mathcal{M}^{1,3}$ and embedded in $\mathbb{R}^6$

Twist Fields Correlators

$$\left\langle \prod_{t=1}^{N_B} \sigma_{M(t)}(x(t)) \right\rangle = \mathcal{N} \left(\{x(t), M(t)\}_{1 \leq t \leq N_B} \right) e^{-S_E(\text{cl}) \left(\{x(t), M(t)\}_{1 \leq t \leq N_B} \right)}$$



D-branes in **factorised** internal space:

- **embedded as lines** in $\mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2$
- **relative rotations** are $SO(2) \simeq U(1)$ elements
- $S_E(\text{cl}) \left(\{x(t), M(t)\}_{1 \leq t \leq N_B} \right) \sim \text{Area} \left(\{f(t), R(t)\}_{1 \leq t \leq N_B} \right)$

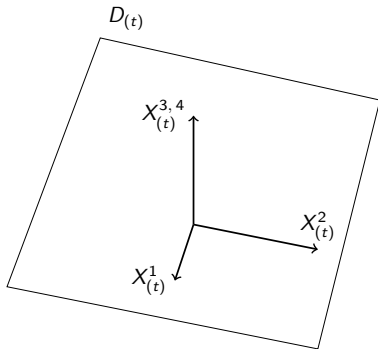
[Cremades, Ibanez, Marchesano (2003); Pesando (2012)]

SO(4) Rotations

Consider $\mathbb{R}^4 \times \mathbb{R}^2$ (focus on $\mathbb{R}^4$):

SO(4) Rotations

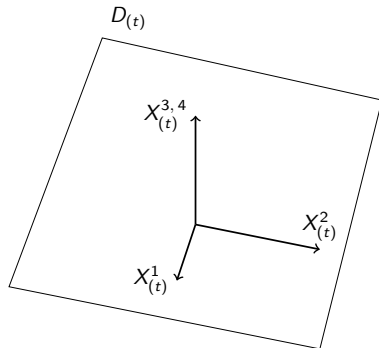
Consider $\mathbb{R}^4 \times \mathbb{R}^2$ (focus on $\mathbb{R}^4$):



$$(X_{(t)})^I = (R_{(t)})^I_J X^J - g_{(t)}^I$$

SO(4) Rotations

Consider $\mathbb{R}^4 \times \mathbb{R}^2$ (focus on $\mathbb{R}^4$):



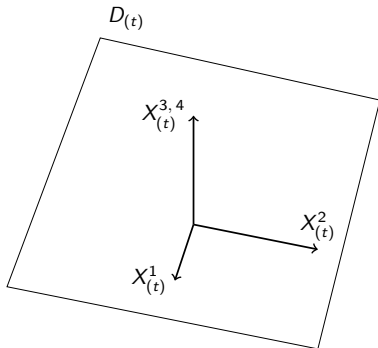
where

$$(X_{(t)})^I = (R_{(t)})^I_J X^J - g_{(t)}^I$$

$$R_{(t)} \in \frac{\text{SO}(4)}{\text{S}(\text{O}(2) \times \text{O}(2))}$$

SO(4) Rotations

Consider $\mathbb{R}^4 \times \mathbb{R}^2$ (focus on $\mathbb{R}^4$):



where

$$(X_{(t)})^I = (R_{(t)})^I_J X^J - g_{(t)}^I$$

that is

$$R_{(t)} \in \frac{\text{SO}(4)}{\text{S}(\text{O}(2) \times \text{O}(2))}$$

$$[R_{(t)}] = \{R_{(t)} \sim \mathcal{O}_{(t)} R_{(t)}\}$$

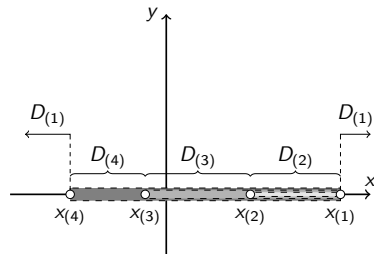
Boundary Conditions

What are the consequences for open strings?

Boundary Conditions

What are the consequences for open strings?

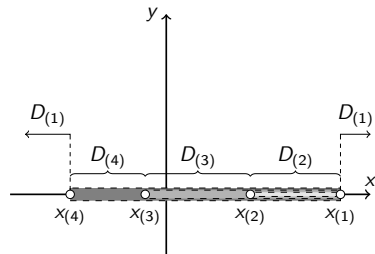
- consider $u = x + iy = e^{\tau_e + i\sigma}$ and $\bar{u} = u^*$
- let $x_{(t)} < x_{(t-1)}$ be the **worldsheet intersection points on real axis**
- $X_{(t)}^{1,2}$ are **Neumann**, $X_{(t)}^{3,4}$ are **Dirichlet**



Boundary Conditions

What are the consequences for open strings?

- consider $u = x + iy = e^{\tau_e + i\sigma}$ and $\bar{u} = u^*$
- let $x_{(t)} < x_{(t-1)}$ be the **worldsheet intersection points on real axis**
- $X_{(t)}^{1,2}$ are **Neumann**, $X_{(t)}^{3,4}$ are **Dirichlet**



Branch Cuts and Discontinuities for $x \in D_{(t)}$

$$\begin{cases} \partial_u X(x + i0^+) = U_{(t)} \cdot \partial_{\bar{u}} \bar{X}(x - i0^+) = \left[R_{(t)}^{-1} \cdot (\sigma_3 \otimes \mathbb{1}_2) \cdot R_{(t)} \right] \cdot \partial_{\bar{u}} \bar{X}(x - i0^+) \\ X(x_{(t)}, x_{(t)}) = f_{(t)} \end{cases}$$

Doubling Trick and Spinor Representation

Doubling Trick

$$\partial_z \mathcal{X}(z) = \begin{cases} \partial_u X(u) & \text{if } z \in \mathcal{H}_{>}^{(\bar{t})} \\ U_{(\bar{t})} \partial_{\bar{u}} \bar{X}(\bar{u}) & \text{if } z \in \mathcal{H}_{<}^{(\bar{t})} \end{cases} \Rightarrow \begin{aligned} \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i} \delta_+) &= \mathcal{U}_{(t, t+1)} \partial_z \mathcal{X}(x_{(t)} + \delta_+), \\ \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i} \delta_-) &= \tilde{\mathcal{U}}_{(t, t+1)} \partial_z \mathcal{X}(x_{(t)} + \delta_-), \end{aligned}$$

where $\mathcal{H}_{\gtrless}^{(t)} = \{z \in \mathbb{C} \mid \text{Im } z \gtrless 0 \text{ or } z \in D_{(t)}\}$ and $\delta_{\pm} = \eta \pm i0^+$.

Doubling Trick and Spinor Representation

Doubling Trick

$$\partial_z \mathcal{X}(z) = \begin{cases} \partial_u X(u) & \text{if } z \in \mathcal{H}_{>}^{(\bar{t})} \\ U_{(\bar{t})} \partial_{\bar{u}} \bar{X}(\bar{u}) & \text{if } z \in \mathcal{H}_{<}^{(\bar{t})} \end{cases} \Rightarrow \begin{aligned} \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i} \delta_+) &= \mathcal{U}_{(t, t+1)} \partial_z \mathcal{X}(x_{(t)} + \delta_+), \\ \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i} \delta_-) &= \tilde{\mathcal{U}}_{(t, t+1)} \partial_z \mathcal{X}(x_{(t)} + \delta_-), \end{aligned}$$

where $\mathcal{H}_{\gtrless}^{(t)} = \{z \in \mathbb{C} \mid \text{Im } z \gtrless 0 \text{ or } z \in D_{(t)}\}$ and $\delta_{\pm} = \eta \pm i0^+$.

Doubling Trick and Spinor Representation

Doubling Trick

$$\partial_z \mathcal{X}(z) = \begin{cases} \partial_u X(u) & \text{if } z \in \mathcal{H}_{>}^{(\bar{t})} \\ U_{(\bar{t})} \partial_{\bar{u}} \bar{X}(\bar{u}) & \text{if } z \in \mathcal{H}_{<}^{(\bar{t})} \end{cases} \Rightarrow \begin{aligned} \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i} \delta_+) &= \mathcal{U}_{(t, t+1)} \partial_z \mathcal{X}(x_{(t)} + \delta_+), \\ \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i} \delta_-) &= \tilde{\mathcal{U}}_{(t, t+1)} \partial_z \mathcal{X}(x_{(t)} + \delta_-), \end{aligned}$$

where $\mathcal{H}_{\gtrless}^{(t)} = \{z \in \mathbb{C} \mid \text{Im } z \gtrless 0 \text{ or } z \in D_{(t)}\}$ and $\delta_{\pm} = \eta \pm i0^+$.

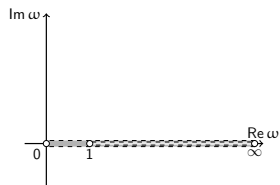
Use Pauli matrices $\tau = (i \mathbb{1}_2, \vec{\sigma})$:

$$\partial_z \mathcal{X}_{(s)}(z) = \partial_z \mathcal{X}'(z) \tau_I \Rightarrow \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i} \delta_{\pm}) = \overset{(\sim)}{\mathcal{L}}_{(t, t+1)} \partial_z \mathcal{X}(x_{(t)} + \delta_{\pm}) \overset{(\sim)}{\mathcal{R}}_{(t, t+1)}$$

where

$$\overset{(\sim)}{\mathcal{L}}_{(t, t+1)} \in \text{SU}(2)_L \quad \text{and} \quad \overset{(\sim)}{\mathcal{R}}_{(t, t+1)} \in \text{SU}(2)_R$$

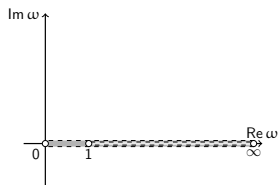
Hypergeometric Basis



Sum over all contributions:

$$\partial_z \mathcal{X}(z) = \sum_{l,r} c_{lr} (-\omega_z)^{A_{lr}} (1 - \omega_z)^{B_{lr}} B_{0,l}^{(L)}(\omega_z) \left(B_{0,r}^{(R)}(\omega_z) \right)^T$$

Hypergeometric Basis



Sum over all contributions:

$$\partial_z \mathcal{X}(z) = \sum_{l,r} c_{lr} (-\omega_z)^{A_{lr}} (1 - \omega_z)^{B_{lr}} B_{0,l}^{(L)}(\omega_z) \left(B_{0,r}^{(R)}(\omega_z) \right)^T$$

Basis of Solutions

$$B_{0,n}(\omega_z) = \begin{pmatrix} 1 & 0 \\ 0 & K_n \end{pmatrix} \begin{pmatrix} \frac{1}{\Gamma(c_n)} {}_2F_1(a_n, b_n; c_n; \omega_z) \\ \frac{(-\omega_z)^{1-c_n}}{\Gamma(2-c_n)} {}_2F_1(a_n + 1 - c_n, b_n + 1 - c_n; 2 - c_n; \omega_z) \end{pmatrix}$$

The Solution

Operations sequence:

1. rotation matrix = monodromy matrix

The Solution

Operations sequence:

1. rotation matrix = monodromy matrix
2. contiguity relations $\Rightarrow$ independent hypergeometrics

The Solution

Operations sequence:

1. rotation matrix = monodromy matrix
2. contiguity relations $\Rightarrow$ independent hypergeometrics
3. finite action $\Rightarrow$ 2 solutions (no. of d.o.f. is correctly saturated)

The Solution

Operations sequence:

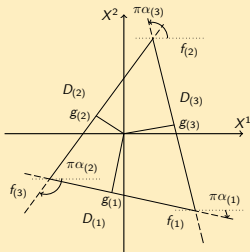
1. rotation matrix = monodromy matrix
2. contiguity relations $\Rightarrow$ independent hypergeometrics
3. finite action $\Rightarrow$ 2 solutions (no. of d.o.f. is correctly saturated)
4. boundary conditions $\Rightarrow$ fix free constants c_{lr}

The Solution

Operations sequence:

1. rotation matrix = monodromy matrix
2. contiguity relations $\Rightarrow$ independent hypergeometrics
3. finite action $\Rightarrow$ 2 solutions (no. of d.o.f. is correctly saturated)
4. boundary conditions $\Rightarrow$ fix free constants c_{lr}

Physical Interpretation



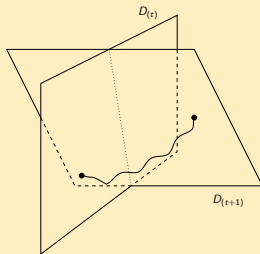
$$S_{\mathbb{R}^4} \Big|_{\text{on-shell}} = \frac{1}{2\pi\alpha'} \sum_{t=1}^3 \left(\frac{1}{2} |g_{(t)}^\perp| |f_{(t-1)} - f_{(t)}| \right) = \text{Area}(\{f_{(t)}\})$$

The Solution

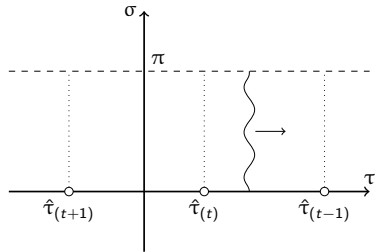
Operations sequence:

1. rotation matrix = monodromy matrix
2. contiguity relations $\Rightarrow$ independent hypergeometrics
3. finite action $\Rightarrow$ 2 solutions (no. of d.o.f. is correctly saturated)
4. boundary conditions $\Rightarrow$ fix free constants c_{lr}

Physical Interpretation



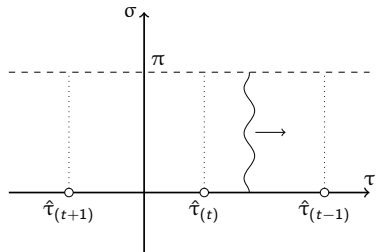
Fermions on the Strip



Action of Boundary Changing Operators

$$\begin{cases} \psi_-^i(\tau, 0) &= (R_{(t)})^I{}_J \psi_+^J(\tau, 0) \quad \text{for } \tau \in (\hat{\tau}_{(t)}, \hat{\tau}_{(t-1)}) \\ \psi_-^I(\tau, \pi) &= -\psi_+^I(\tau, \pi) \quad \text{for } \tau \in \mathbb{R} \end{cases}$$

Fermions on the Strip



Action of Boundary Changing Operators

$$\begin{cases} \psi_-^i(\tau, 0) &= (R_{(t)})^I{}_J \psi_+^J(\tau, 0) \quad \text{for } \tau \in (\hat{\tau}_{(t)}, \hat{\tau}_{(t-1)}) \\ \psi_-^I(\tau, \pi) &= -\psi_+^I(\tau, \pi) \quad \text{for } \tau \in \mathbb{R} \end{cases}$$

Stress-energy Tensor

$$\mathcal{T}_{\pm\pm}(\xi_{\pm}) = -i \frac{T}{4} \psi_{\pm, I}^*(\xi_{\pm}) \overset{\leftrightarrow}{\partial} \psi_{\pm}^I(\xi_{\pm}) \quad \Rightarrow \quad \begin{cases} \dot{H}(\tau) &= 0 \\ \dot{P}(\tau) &\neq 0 \end{cases} \Leftrightarrow \tau \in (\tau_{(t)}, \tau_{(t-1)})$$

Conserved Product and Operators

Expand on a basis of solutions

$$\psi_{\pm}(\xi_{\pm}) = \sum_{n=-\infty}^{+\infty} b_n \psi_n(\xi_{\pm}) \quad \Rightarrow \quad \Psi(z) = \begin{cases} \psi_{E,+}(u) & \text{if } z \in \mathcal{H}_{>}^{(\bar{t})} \\ \psi_{E,-}(u) & \text{if } z \in \mathcal{H}_{<}^{(\bar{t})} \end{cases}$$

Conserved Product and Operators

Expand on a basis of solutions

$$\psi_{\pm}(\xi_{\pm}) = \sum_{n=-\infty}^{+\infty} b_n \psi_n(\xi_{\pm}) \quad \Rightarrow \quad \Psi(z) = \begin{cases} \psi_{E,+}(u) & \text{if } z \in \mathcal{H}_{>}^{(\bar{t})} \\ \psi_{E,-}(u) & \text{if } z \in \mathcal{H}_{<}^{(\bar{t})} \end{cases}$$

Conserved Product and Dual Basis

$$\langle\langle {}^*\psi_n, \psi_m \rangle = 2\pi\mathcal{N} \oint \frac{dz}{2\pi i} {}^*\Psi_n {}^*\Psi_m = \delta_{n,m} \quad \Rightarrow \quad \langle\langle {}^*\Psi_n^{(*)}, \Psi^{(*)} \rangle = b_n^{(\dagger)}$$

Conserved Product and Operators

Expand on a basis of solutions

$$\psi_{\pm}(\xi_{\pm}) = \sum_{n=-\infty}^{+\infty} b_n \psi_n(\xi_{\pm}) \quad \Rightarrow \quad \Psi(z) = \begin{cases} \psi_{E,+}(u) & \text{if } z \in \mathcal{H}_{>}^{(\bar{t})} \\ \psi_{E,-}(u) & \text{if } z \in \mathcal{H}_{<}^{(\bar{t})} \end{cases}$$

Conserved Product and Dual Basis

$$\langle\langle {}^*\psi_n, \psi_m \rangle\rangle = 2\pi\mathcal{N} \oint \frac{dz}{2\pi i} {}^*\Psi_n {}^*\Psi_m = \delta_{n,m} \quad \Rightarrow \quad \langle\langle {}^*\Psi_n^{(*)}, \Psi^{(*)} \rangle\rangle = b_n^{(\dagger)}$$

Derive the algebra of operators:

$$[b_n, b_m^{\dagger}]_+ = \frac{2\mathcal{N}}{T} \langle\langle {}^*\Psi_n^*, \Psi_m^* \rangle\rangle$$

Twisted Complex Fermions

Consider the case $R_{(t)} = e^{i\pi\alpha_{(t)}} \in U(1)$:

$$\Psi(x_{(t)} + e^{2\pi i}\delta) = e^{i\pi\epsilon_{(t)}} \Psi(x_{(t)} + \delta)$$

where

$$\epsilon_{(t)} = \alpha_{(t+1)} - \alpha_{(t)} + \theta(\alpha_{(t)} - \alpha_{(t+1)} - 1) - \theta(\alpha_{(t+1)} - \alpha_{(t)} - 1)$$

Twisted Complex Fermions

Consider the case $R_{(t)} = e^{i\pi\alpha_{(t)}} \in U(1)$:

$$\Psi(x_{(t)} + e^{2\pi i}\delta) = e^{i\pi\epsilon_{(t)}} \Psi(x_{(t)} + \delta)$$

where

$$\epsilon_{(t)} = \alpha_{(t+1)} - \alpha_{(t)} + \theta(\alpha_{(t)} - \alpha_{(t+1)} - 1) - \theta(\alpha_{(t+1)} - \alpha_{(t)} - 1)$$

Basis of Solutions

$$\Psi_n(z; \{x_{(t)}\}) = \mathcal{N}_\Psi z^{-n} \prod_{t=1}^N \left(1 - \frac{z}{x_{(t)}}\right)^{n_{(t)} + \frac{\epsilon_{(t)}}{2}}$$

$$^*\Psi_n(z; \{x_{(t)}\}) = \frac{1}{2\pi\mathcal{N}\mathcal{N}_\Psi} z^{n-1} \prod_{t=1}^N \left(1 - \frac{z}{x_{(t)}}\right)^{-\tilde{n}_{(t)} + \frac{\epsilon_{(t)}}{2}}$$

Vacua

Define the **vacuum** with respect to b_n :

$$b_n \left| \{x_{(t)}\} \right\rangle = 0 \quad \text{for } n \geq 1$$

$$b_n \left| \tilde{0} \right\rangle = 0 \quad \text{for } n \geq n_{(t)} + \frac{\epsilon_{(t)}}{2} + \frac{1}{2}$$

Vacua

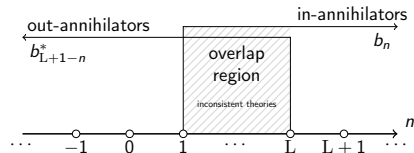
Define the **vacuum** with respect to b_n :

$$b_n \left| \{x_{(t)}\} \right\rangle = 0 \quad \text{for } n \geq 1$$

$$b_n \left| \tilde{0} \right\rangle = 0 \quad \text{for } n \geq n_{(t)} + \frac{\epsilon_{(t)}}{2} + \frac{1}{2}$$

Theories are subject to consistency conditions:

$$L = n_{(t)} + \tilde{n}_{(t)}$$



Vacua

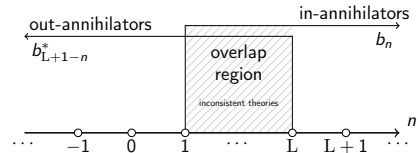
Define the **vacuum** with respect to b_n :

$$b_n \left| \{x_{(t)}\} \right\rangle = 0 \quad \text{for } n \geq 1$$

$$b_n \left| \tilde{0} \right\rangle = 0 \quad \text{for } n \geq n_{(t)} + \frac{\epsilon_{(t)}}{2} + \frac{1}{2}$$

Theories are subject to consistency conditions:

$$L = n_{(t)} + \tilde{n}_{(t)} = 0$$



Stress-energy Tensor and CFT Approach

Compute the OPEs leading to the stress-energy tensor:

$$\mathcal{T}(z) = \frac{\pi T}{2} \mathcal{N}_{\Psi}^2 \sum_{n, m=-\infty}^{+\infty} : b_n b_m^* : z^{-n-m} \left[\frac{m-n}{2} + 2 \sum_{t=1}^N \frac{n_{(t)} + \frac{\epsilon_{(t)}}{2}}{z - x_{(t)}} \right] + \frac{1}{2} \left(\sum_{t=1}^N \frac{n_{(t)} + \frac{\epsilon_{(t)}}{2}}{z - x_{(t)}} \right)^2$$

Stress-energy Tensor and CFT Approach

Compute the OPEs leading to the stress-energy tensor:

$$\mathcal{T}(z) = \frac{\pi T}{2} \mathcal{N}_{\Psi}^2 \sum_{n, m=-\infty}^{+\infty} : b_n b_m^* : z^{-n-m} \left[\frac{m-n}{2} + 2 \sum_{t=1}^N \frac{n_{(t)} + \frac{\epsilon_{(t)}}{2}}{z - x_{(t)}} \right] + \frac{1}{2} \left(\sum_{t=1}^N \frac{n_{(t)} + \frac{\epsilon_{(t)}}{2}}{z - x_{(t)}} \right)^2$$

Invariant Vacuum and Spin Fields

$$|\{x_{(t)}\}\rangle = \mathcal{N}(\{x_{(t)}\}) \mathcal{R} \left[\prod_{t=1}^M S_{(t)}(x_{(t)}) \right] |0\rangle_{\text{SL}_2(\mathbb{R})}$$

Spin Fields Amplitudes

Equivalence with Bosonization

$$\partial_{x_{(t)}} \langle \{x_{(t)}\} | \{x_{(t)}\} \rangle = \oint_{x_{(t)}} \frac{dz}{2\pi i} \frac{\langle \{x_{(t)}\} | \mathcal{T}(z) | \{x_{(t)}\} \rangle}{\langle \{x_{(t)}\} | \{x_{(t)}\} \rangle}$$

$$\Rightarrow \langle \{x_{(t)}\} | \{x_{(t)}\} \rangle = \mathcal{N}(\{\epsilon_{(t)}\}) \prod_{\substack{t=1 \\ t>u}}^N (x_{(u)} - x_{(t)})^{\left(n_{(u)} + \frac{\epsilon_{(u)}}{2}\right) \left(n_{(t)} + \frac{\epsilon_{(t)}}{2}\right)}$$

Spin Fields Amplitudes

Equivalence with Bosonization

$$\partial_{x_{(t)}} \langle \{x_{(t)}\} | \{x_{(t)}\} \rangle = \oint_{x_{(t)}} \frac{dz}{2\pi i} \frac{\langle \{x_{(t)}\} | \mathcal{T}(z) | \{x_{(t)}\} \rangle}{\langle \{x_{(t)}\} | \{x_{(t)}\} \rangle}$$

$$\Rightarrow \langle \{x_{(t)}\} | \{x_{(t)}\} \rangle = \mathcal{N}(\{\epsilon_{(t)}\}) \prod_{\substack{t=1 \\ t>u}}^N (x_{(u)} - x_{(t)})^{\left(n_{(u)} + \frac{\epsilon_{(u)}}{2}\right) \left(n_{(t)} + \frac{\epsilon_{(t)}}{2}\right)}$$

- (semi-)phenomenological models involve **twist and spin** fields and **open strings**

Spin Fields Amplitudes

Equivalence with Bosonization

$$\partial_{x_{(t)}} \langle \{x_{(t)}\} | \{x_{(t)}\} \rangle = \oint_{x_{(t)}} \frac{dz}{2\pi i} \frac{\langle \{x_{(t)}\} | \mathcal{T}(z) | \{x_{(t)}\} \rangle}{\langle \{x_{(t)}\} | \{x_{(t)}\} \rangle}$$

$$\Rightarrow \langle \{x_{(t)}\} | \{x_{(t)}\} \rangle = \mathcal{N}(\{\epsilon_{(t)}\}) \prod_{\substack{t=1 \\ t>u}}^N (x_{(u)} - x_{(t)})^{\left(n_{(u)} + \frac{\epsilon_{(u)}}{2}\right) \left(n_{(t)} + \frac{\epsilon_{(t)}}{2}\right)}$$

- (semi-)phenomenological models involve **twist and spin** fields and **open strings**
- general framework for **bosonic** open strings with **intersecting D-branes**

Spin Fields Amplitudes

Equivalence with Bosonization

$$\partial_{x_{(t)}} \langle \{x_{(t)}\} | \{x_{(t)}\} \rangle = \oint_{x_{(t)}} \frac{dz}{2\pi i} \frac{\langle \{x_{(t)}\} | \mathcal{T}(z) | \{x_{(t)}\} \rangle}{\langle \{x_{(t)}\} | \{x_{(t)}\} \rangle}$$

$$\Rightarrow \langle \{x_{(t)}\} | \{x_{(t)}\} \rangle = \mathcal{N}(\{\epsilon_{(t)}\}) \prod_{\substack{t=1 \\ t > u}}^N (x_{(u)} - x_{(t)})^{\left(n_{(u)} + \frac{\epsilon_{(u)}}{2}\right) \left(n_{(t)} + \frac{\epsilon_{(t)}}{2}\right)}$$

- (semi-)phenomenological models involve **twist and spin** fields and **open strings**
- general framework for **bosonic** open strings with **intersecting D-branes**
- leading contribution for **twist fields**

Spin Fields Amplitudes

Equivalence with Bosonization

$$\partial_{x_{(t)}} \langle \{x_{(t)}\} | \{x_{(t)}\} \rangle = \oint_{x_{(t)}} \frac{dz}{2\pi i} \frac{\langle \{x_{(t)}\} | \mathcal{T}(z) | \{x_{(t)}\} \rangle}{\langle \{x_{(t)}\} | \{x_{(t)}\} \rangle}$$

$$\Rightarrow \langle \{x_{(t)}\} | \{x_{(t)}\} \rangle = \mathcal{N}(\{\epsilon_{(t)}\}) \prod_{\substack{t=1 \\ t>u}}^N (x_{(u)} - x_{(t)})^{\left(n_{(u)} + \frac{\epsilon_{(u)}}{2}\right) \left(n_{(t)} + \frac{\epsilon_{(t)}}{2}\right)}$$

- (semi-)phenomenological models involve **twist and spin** fields and **open strings**
- general framework for **bosonic** open strings with **intersecting D-branes**
- leading contribution for **twist fields**
- **spin fields** as **boundary changing operators** on **defects**

Spin Fields Amplitudes

Equivalence with Bosonization

$$\partial_{x_{(t)}} \langle \{x_{(t)}\} | \{x_{(t)}\} \rangle = \oint_{x_{(t)}} \frac{dz}{2\pi i} \frac{\langle \{x_{(t)}\} | \mathcal{T}(z) | \{x_{(t)}\} \rangle}{\langle \{x_{(t)}\} | \{x_{(t)}\} \rangle}$$

$$\Rightarrow \langle \{x_{(t)}\} | \{x_{(t)}\} \rangle = \mathcal{N}(\{\epsilon_{(t)}\}) \prod_{\substack{t=1 \\ t > u}}^N (x_{(u)} - x_{(t)})^{\left(n_{(u)} + \frac{\epsilon_{(u)}}{2}\right) \left(n_{(t)} + \frac{\epsilon_{(t)}}{2}\right)}$$

- (semi-)phenomenological models involve **twist and spin** fields and **open strings**
- general framework for **bosonic** open strings with **intersecting D-branes**
- leading contribution for **twist fields**
- **spin fields** as **boundary changing operators** on **defects**
- alternative framework for amplitudes (extension to (non) Abelian twist/spin fields?)

Contents

Conformal Symmetry and Geometry of the Worldsheet

Preliminary Concepts and Tools

D-branes Intersecting at Angles

Fermions and Point-like Defect CFT

Cosmological Backgrounds and Divergences

Orbifolds and Cosmological Toy Models

Null Boost Orbifold

Deep Learning the Geometry of String Theory

Machine Learning and Deep Learning

Machine Learning for String Theory

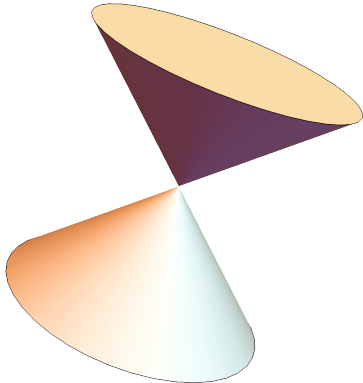
AI Implementations for Geometry and Strings

A Few Words on a Theory of Everything

string theory = theory of everything = nuclear forces + gravity

A Few Words on a Theory of Everything

string theory = theory of everything = nuclear forces + gravity

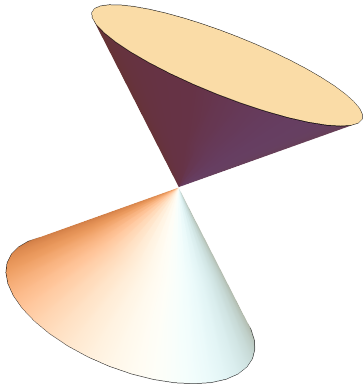


From the phenomenological point of view:

- cosmological implications

A Few Words on a Theory of Everything

string theory = theory of everything = nuclear forces + gravity

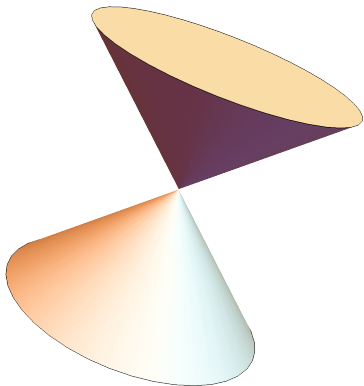


From the phenomenological point of view:

- cosmological implications
- Big Bang(-like) singularities

A Few Words on a Theory of Everything

string theory = theory of everything = nuclear forces + gravity

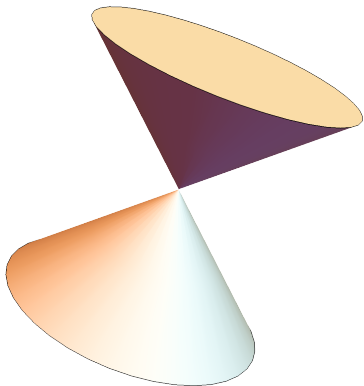


From the phenomenological point of view:

- cosmological implications
- Big Bang(-like) singularities
- toy models of **space-like singularities**

A Few Words on a Theory of Everything

string theory = theory of everything = nuclear forces + gravity



From the phenomenological point of view:

- cosmological implications
- Big Bang(-like) singularities
- toy models of **space-like singularities**



time-dependent orbifold models

[Craps, Kutasov, Rajesh (2002); Liu, Moore, Seiberg (2002)]

Orbifolds

Mathematics

- manifold M
- (Lie) group G
- *stabilizer*: $G_p = \{g \in G \mid gp = p \in M\}$
- *orbit*: $Gp = \{gp \in M \mid g \in G\}$
- charts $\phi = \pi \circ \mathcal{P}$ where:
 - $\mathcal{P}: U \subset \mathbb{R}^n \rightarrow U/G$
 - $\pi: U/G \rightarrow M$

$\Rightarrow$

Physics

- global orbit space M/G
- G group of isometries
- fixed points
- additional d.o.f. (*twisted states*)
- singular limits of CY manifolds

Orbifolds

Mathematics

- manifold M
- (Lie) group G
- *stabilizer*: $G_p = \{g \in G \mid gp = p \in M\}$
- *orbit*: $Gp = \{gp \in M \mid g \in G\}$
- charts $\phi = \pi \circ \mathcal{P}$ where:
 - $\mathcal{P}: U \subset \mathbb{R}^n \rightarrow U/G$
 - $\pi: U/G \rightarrow M$

$\Rightarrow$

Physics

- global orbit space M/G
- G group of isometries
- fixed points
- additional d.o.f. (*twisted states*)
- singular limits of CY manifolds

Use **time-dependent orbifolds** to model singularities in time

Cosmological Singularities

Use **time-dependent orbifolds** to model **space-like singularities**:

divergent closed string amplitudes $\Rightarrow$ gravitational backreaction?

Cosmological Singularities

Use **time-dependent orbifolds** to model **space-like singularities**:

divergent closed string amplitudes $\Rightarrow$ gravitational backreaction?

Divergences

Even in simple models (e.g. NBO, more on this later) the 4 tachyons amplitude is divergent **at tree level**:

$$A_4 \sim \int_{q \sim \infty} \frac{dq}{|q|} \mathcal{A}(q)$$

where

$$\mathcal{A}_{\text{closed}}(q) \sim q^{4-\alpha' \|\vec{p}_\perp\|^2} \quad \text{and} \quad \mathcal{A}_{\text{open}}(q) \sim q^{1-\alpha' \|\vec{p}_\perp\|^2} \text{tr}([T_1, T_2]_+ [T_3, T_4]_+)$$

Null Boost Orbifold

Start from $(x^+, x^-, x^2, \vec{x}) \in \mathcal{M}^{1,D-1}$:

$$\begin{cases} u &= x^- \\ z &= \frac{x^2}{\Delta x^-} \\ v &= x^+ - \frac{1}{2} \frac{(x^2)^2}{x^-} \end{cases} \Rightarrow ds^2 = -2 du dv + (\Delta u)^2 dz^2 + \delta_{ij} dx^i dx^j$$

Null Boost Orbifold

Start from $(x^+, x^-, x^2, \vec{x}) \in \mathcal{M}^{1,D-1}$:

$$\begin{cases} u &= x^- \\ z &= \frac{x^2}{\Delta x^-} \\ v &= x^+ - \frac{1}{2} \frac{(x^2)^2}{x^-} \end{cases} \Rightarrow ds^2 = -2 du dv + (\Delta u)^2 dz^2 + \delta_{ij} dx^i dx^j$$

Killing Vector and Null Boost Orbifold

$$\kappa = -i(2\pi\Delta)J_{+2} = 2\pi\partial_z \Rightarrow z \sim z + 2\pi n$$

Null Boost Orbifold

Start from $(x^+, x^-, x^2, \vec{x}) \in \mathcal{M}^{1,D-1}$:

$$\begin{cases} u &= x^- \\ z &= \frac{x^2}{\Delta x^-} \\ v &= x^+ - \frac{1}{2} \frac{(x^2)^2}{x^-} \end{cases} \Rightarrow ds^2 = -2 du dv + (\Delta u)^2 dz^2 + \delta_{ij} dx^i dx^j$$

Killing Vector and Null Boost Orbifold

$$\kappa = -i(2\pi\Delta)J_{+2} = 2\pi\partial_z \Rightarrow z \sim z + 2\pi n$$

Consider scalar QED:

$$\Phi_{\{k_+, l, \vec{k}, r\}}(u, v, z, \vec{x}) = e^{i(k_+ v + l z + \vec{k} \cdot \vec{x})} \tilde{\Phi}_{\{k_+, l, \vec{k}, r\}}(u) = \frac{e^{i(k_+ v + l z + \vec{k} \cdot \vec{x})}}{\sqrt{(2\pi)^D |2\Delta k_+|}} e^{-i \frac{l^2}{2\Delta^2 k_+} \frac{1}{u} + i \frac{\|\vec{k}\|^2 + r}{2k_+} u}$$

Scalar QED Interactions

Scalar-photon interactions:

$$S_{\text{sQED}}^{(\text{int})} = \int_{\Omega} d^D x \sqrt{-g} \left(-i e g^{\alpha\beta} a_{\alpha} (\phi^* \partial_{\beta} \phi - \partial_{\beta} \phi^* \phi) + e^2 g^{\alpha\beta} a_{\alpha} a_{\beta} |\phi|^2 - \frac{g^4}{4} |\phi|^4 \right)$$

Scalar QED Interactions

Scalar-photon interactions:

$$S_{\text{sQED}}^{(\text{int})} = \int_{\Omega} d^D x \sqrt{-g} \left(-i e g^{\alpha\beta} a_{\alpha} (\phi^* \partial_{\beta} \phi - \partial_{\beta} \phi^* \phi) + e^2 g^{\alpha\beta} a_{\alpha} a_{\beta} |\phi|^2 - \frac{g^4}{4} |\phi|^4 \right)$$

Terms involved:

$$\mathcal{I}_{\{N\}}^{[\nu]} = \int_{-\infty}^{+\infty} du |\Delta u| u^{\nu} \prod_{i=1}^N \tilde{\Phi}_{\{k_{+(i)}, l_{(i)}, \vec{k}_{(i)}, r_{(i)}\}}(u)$$

$$\mathcal{J}_{\{N\}}^{[\nu]} = \int_{-\infty}^{+\infty} du |\Delta| |u|^{1+\nu} \prod_{i=1}^N \tilde{\Phi}_{\{k_{+(i)}, l_{(i)}, \vec{k}_{(i)}, r_{(i)}\}}(u)$$

Scalar QED Interactions

Scalar-photon interactions:

$$S_{\text{sQED}}^{(\text{int})} = \int_{\Omega} d^D x \sqrt{-g} \left(-i e g^{\alpha\beta} a_{\alpha} (\phi^* \partial_{\beta} \phi - \partial_{\beta} \phi^* \phi) + e^2 g^{\alpha\beta} a_{\alpha} a_{\beta} |\phi|^2 - \frac{g^4}{4} |\phi|^4 \right)$$

Terms involved:

$$\mathcal{I}_{\{N\}}^{[\nu]} = \int_{-\infty}^{+\infty} du |\Delta u| u^{\nu} \prod_{i=1}^N \tilde{\Phi}_{\{k_{+(i)}, l_{(i)}, \vec{k}_{(i)}, r_{(i)}\}}(u)$$

$$\mathcal{J}_{\{N\}}^{[\nu]} = \int_{-\infty}^{+\infty} du |\Delta| |u|^{1+\nu} \prod_{i=1}^N \tilde{\Phi}_{\{k_{+(i)}, l_{(i)}, \vec{k}_{(i)}, r_{(i)}\}}(u)$$

most terms **do not converge** due to **isolated zeros** ($l_{(*)} \equiv 0$) and cannot be recovered even with a **distributional interpretations** due to the term $\propto u^{-1}$ in the exponential

String and Field Theory

So far:

- field theory presents **divergences** (even sQED $\rightarrow$ eikonal?)

String and Field Theory

So far:

- field theory presents **divergences** (even sQED $\rightarrow$ eikonal?)
- divergences are **not (only) gravitational**

String and Field Theory

So far:

- field theory presents **divergences** (even sQED $\rightarrow$ eikonal?)
- divergences are **not (only) gravitational**
- **vanishing volume** in phase space of the compact direction is responsible for the divergence

String and Field Theory

So far:

- field theory presents **divergences** (even sQED $\rightarrow$ eikonal?)
- divergences are **not (only) gravitational**
- **vanishing volume** in phase space of the compact direction is responsible for the divergence

What about string theory?

String and Field Theory

So far:

- field theory presents **divergences** (even sQED $\rightarrow$ eikonal?)
- divergences are **not (only) gravitational**
- **vanishing volume** in phase space of the compact direction is responsible for the divergence

What about string theory?

Massive String States

$$V_M(x; k, S, \xi) =: \left(\frac{i}{\sqrt{2\alpha'}} \xi \cdot \partial_x^2 X(x, x) + \left(\frac{i}{\sqrt{2\alpha'}} \right)^2 S_{\alpha\beta} \partial_x X^\alpha(x, x) \partial_x X^\beta(x, x) \right) e^{ik \cdot X(x, x)}:$$

String and Field Theory

So far:

- field theory presents **divergences** (even sQED $\rightarrow$ eikonal?)
- divergences are **not (only) gravitational**
- **vanishing volume** in phase space of the compact direction is responsible for the divergence

What about string theory?

Massive String States

$$V_M(x; k, S, \xi) =: \left(\frac{i}{\sqrt{2\alpha'}} \xi \cdot \partial_x^2 X(x, x) + \left(\frac{i}{\sqrt{2\alpha'}} \right)^2 S_{\alpha\beta} \partial_x X^\alpha(x, x) \partial_x X^\beta(x, x) \right) e^{ik \cdot X(x, x)}:$$

*string theory cannot do **better than field theory** (EFT) if the latter **does not exist** (even a Wilson line around z does not prevent such behaviour)*

Resolution and Motivation

Introduce the generalised NBO:

$$\begin{cases} u &= x^- \\ z &= \frac{1}{2x^-} \left(\frac{x^2}{\Delta_2} + \frac{x^3}{\Delta_3} \right) \\ w &= \frac{1}{2x^-} \left(\frac{x^2}{\Delta_2} - \frac{x^3}{\Delta_3} \right) \\ v &= x^+ - \frac{1}{2x^-} \left((x^2)^2 + (x^3)^2 \right) \end{cases} \quad \Rightarrow \quad \kappa = -2\pi i (\Delta_2 J_{+2} + \Delta_3 J_{+3}) = 2\pi \partial_z$$

Resolution and Motivation

Introduce the **generalised NBO**:

$$\begin{cases} u &= x^- \\ z &= \frac{1}{2x^-} \left(\frac{x^2}{\Delta_2} + \frac{x^3}{\Delta_3} \right) \\ w &= \frac{1}{2x^-} \left(\frac{x^2}{\Delta_2} - \frac{x^3}{\Delta_3} \right) \\ v &= x^+ - \frac{1}{2x^-} \left((x^2)^2 + (x^3)^2 \right) \end{cases} \Rightarrow \kappa = -2\pi i (\Delta_2 J_{+2} + \Delta_3 J_{+3}) = 2\pi \partial_z$$

Distributional Interpretation

$$\tilde{\Phi}_{\{k_+, p, l, \vec{k}, r\}}(u) = \frac{1}{2\sqrt{(2\pi)^D |\Delta_2 \Delta_3 k_+|}} \frac{1}{|u|} e^{-i \left(\frac{1}{8k_+ u} \left[\frac{(l+p)^2}{\Delta_2^2} + \frac{(l-p)^2}{\Delta_3^2} \right] - \frac{\|\vec{k}\|^2 + r}{2k_+} u \right)}$$

On the Divergences and Their Nature

- divergences are present in sQED and **open string** sector

On the Divergences and Their Nature

- divergences are present in sQED and **open string** sector
- singularities $\Rightarrow$ **massive states** are no longer spectators

On the Divergences and Their Nature

- divergences are present in sQED and **open string** sector
- singularities $\Rightarrow$ **massive states** are no longer spectators
- vanishing volume (**compact orbifold directions**) $\Rightarrow$ particles “cannot escape”

On the Divergences and Their Nature

- divergences are present in sQED and **open string** sector
- singularities $\Rightarrow$ **massive states** are no longer spectators
- vanishing volume (**compact orbifold directions**) $\Rightarrow$ particles “cannot escape”
- **non compact** orbifold directions $\Rightarrow$ interpretation of **amplitudes as distributions**

On the Divergences and Their Nature

- divergences are present in sQED and **open string** sector
- singularities $\Rightarrow$ **massive states** are no longer spectators
- vanishing volume (**compact orbifold directions**) $\Rightarrow$ particles “cannot escape”
- **non compact** orbifold directions $\Rightarrow$ interpretation of **amplitudes as distributions**
- issue not restricted to NBO/GNBO but also BO, null brane, etc. (it is a **general issues** connected to the geometry of the underlying space)

On the Divergences and Their Nature

- divergences are present in sQED and **open string** sector
- singularities $\Rightarrow$ **massive states** are no longer spectators
- vanishing volume (**compact orbifold directions**) $\Rightarrow$ particles “cannot escape”
- **non compact** orbifold directions $\Rightarrow$ interpretation of **amplitudes as distributions**
- issue not restricted to NBO/GNBO but also BO, null brane, etc. (it is a **general issues** connected to the geometry of the underlying space)

*spacetime singularities are **hidden into contact terms** and interactions with **massive states**
(the gravitational eikonal deals with massless interactions)*

Contents

Conformal Symmetry and Geometry of the Worldsheet

Preliminary Concepts and Tools

D-branes Intersecting at Angles

Fermions and Point-like Defect CFT

Cosmological Backgrounds and Divergences

Orbifolds and Cosmological Toy Models

Null Boost Orbifold

Deep Learning the Geometry of String Theory

Machine Learning and Deep Learning

Machine Learning for String Theory

AI Implementations for Geometry and Strings

The Simplest Calabi–Yau

Focus on Calabi–Yau 3-folds:

$$h^{r,s} = \dim_{\mathbb{C}} H_{\bar{\partial}}^{r,s}(M, \mathbb{C}) \quad \Rightarrow \quad \begin{cases} h^{0,0} & = h^{3,0} = 1 \\ h^{r,0} & = 0 \quad \text{if } r \neq 3 \\ h^{r,s} & = h^{3-r,3-s} \\ h^{1,1}, h^{2,1} \in \mathbb{N} \end{cases}$$

The Simplest Calabi–Yau

Focus on Calabi–Yau 3-folds:

$$h^{r,s} = \dim_{\mathbb{C}} H_{\bar{\partial}}^{r,s}(M, \mathbb{C}) \quad \Rightarrow \quad \begin{cases} h^{0,0} & = h^{3,0} = 1 \\ h^{r,0} & = 0 \quad \text{if } r \neq 3 \\ h^{r,s} & = h^{3-r,3-s} \\ h^{1,1}, h^{2,1} & \in \mathbb{N} \end{cases}$$

Complete Intersection Calabi–Yau Manifolds

Intersection of hypersurfaces in

$$\mathcal{A} = \mathbb{P}^{n_1} \times \dots \times \mathbb{P}^{n_m}$$

where

$$\mathbb{P}^n: \quad \begin{cases} p_a(Z^0, \dots, Z^n) & = P_{l_1 \dots l_a} Z^{l_1} \dots Z^{l_a} = 0 \\ p_a(\lambda Z^0, \dots, \lambda Z^n) & = \lambda^a p_a(Z^0, \dots, Z^n) \end{cases}$$

[Green, Hübsch (1987); Hübsch (1992)]

Representation of the Output

CICY can be generalised to m projective spaces and k equations. The problem is thus mapped to:

$$\mathcal{R}: \mathbb{Z}^{m \times k} \longrightarrow \mathbb{N}$$

$$\left[\begin{array}{c|ccc} \mathbb{P}^{n_1} & a_1^1 & \dots & a_k^1 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{P}^{n_m} & a_1^m & \dots & a_k^m \end{array} \right] \longrightarrow h^{1,1} \text{ or } h^{2,1}$$

Representation of the Output

CICY can be generalised to m projective spaces and k equations. The problem is thus mapped to:

$$\mathcal{R}: \mathbb{Z}^{m \times k} \longrightarrow \mathbb{N}$$

$$\left[\begin{array}{c|ccc} \mathbb{P}^{n_1} & a_1^1 & \dots & a_k^1 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{P}^{n_m} & a_1^m & \dots & a_k^m \end{array} \right] \longrightarrow h^{1,1} \text{ or } h^{2,1}$$

Machine Learning Approach

What is $\mathcal{R}$?

$$\mathcal{R}(M) \longrightarrow \mathcal{R}_n(M; w) \quad \text{s.t.} \quad \lim_{n \rightarrow \infty} |\mathcal{R}(M) - \mathcal{R}_n(M; w)| = 0$$

Machine Learning

- exchange **analytical solution** with **optimisation problem**

Machine Learning

- exchange **analytical solution** with **optimisation problem**
- use **various algorithms** and exploit **large datasets**

Machine Learning

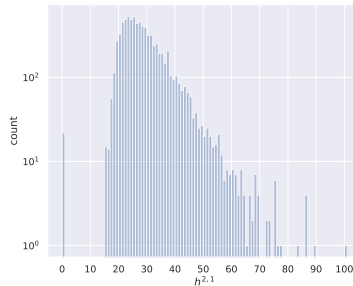
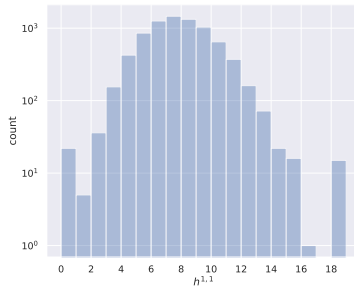
- exchange **analytical solution** with **optimisation problem**
- use **various algorithms** and exploit **large datasets**
- learn a **representation** rather than a **solution**

Machine Learning

- exchange **analytical solution** with **optimisation problem**
- use **various algorithms** and exploit **large datasets**
- learn a **representation** rather than a **solution**
- effectively use knowledge from **computer science, mathematics and physics** to solve problems

Machine Learning

- exchange **analytical solution** with **optimisation problem**
- use **various algorithms** and exploit **large datasets**
- learn a **representation** rather than a **solution**
- effectively use knowledge from **computer science, mathematics and physics** to solve problems



Exploratory Data Analysis

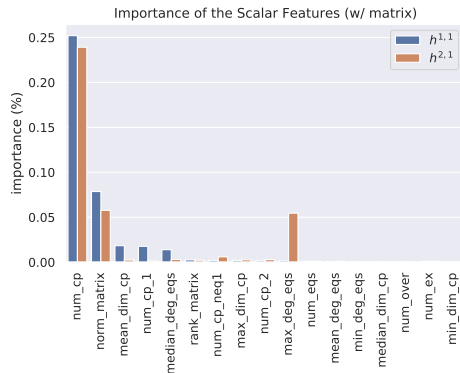
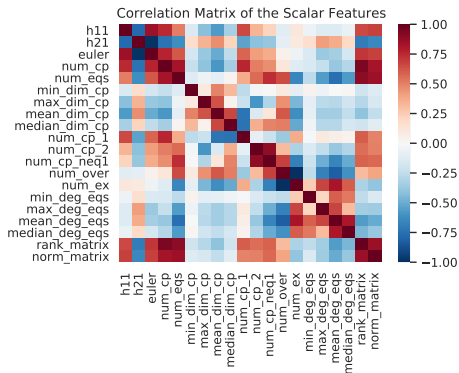
Machine Learning pipeline:

exploratory data analysis → feature **selection** → Hodge numbers

Exploratory Data Analysis

Machine Learning pipeline:

exploratory data analysis → feature selection → Hodge numbers



Dataset

- 7890 CICY manifolds (full dataset)

Dataset

- 7890 CICY manifolds (full dataset)
- **dataset pruning**: no product spaces, no “very far” outliers (reduction of 0.49%)

Dataset

- 7890 CICY manifolds (full dataset)
- **dataset pruning**: no product spaces, no “very far” outliers (reduction of 0.49%)
- $h^{1,1} \in [1, 16]$ and $h^{2,1} \in [15, 86]$

Dataset

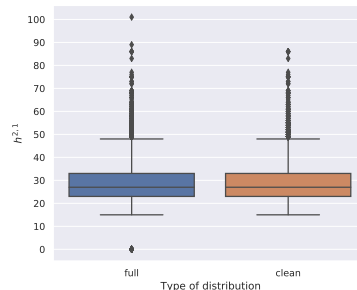
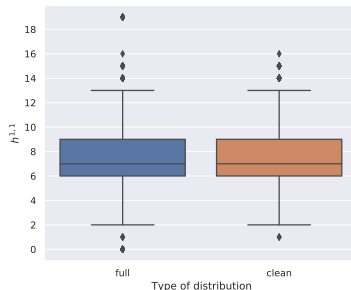
- 7890 CICY manifolds (full dataset)
- **dataset pruning**: no product spaces, no “very far” outliers (reduction of 0.49%)
- $h^{1,1} \in [1, 16]$ and $h^{2,1} \in [15, 86]$
- 80% training, 10% validation, 10% test

Dataset

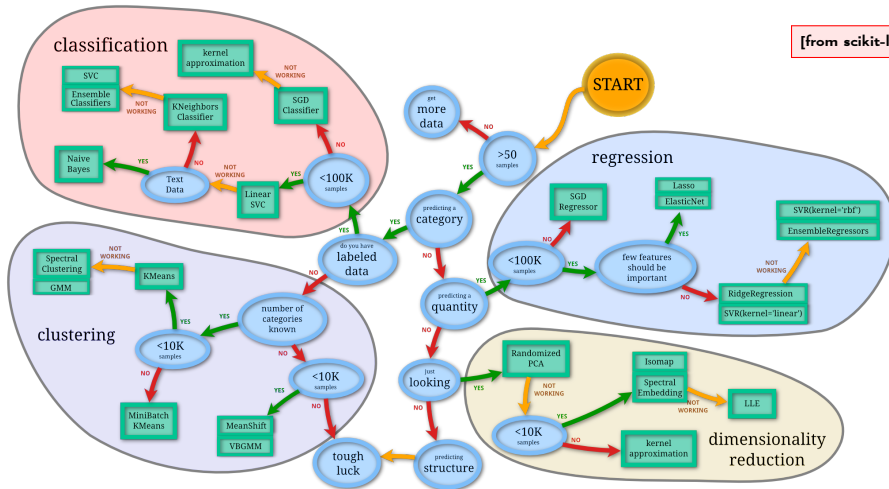
- 7890 CICY manifolds (full dataset)
- **dataset pruning**: no product spaces, no “very far” outliers (reduction of 0.49%)
- $h^{1,1} \in [1, 16]$ and $h^{2,1} \in [15, 86]$
- 80% training, 10% validation, 10% test
- choose **regression**, but evaluate using **accuracy** (round the result)

Dataset

- 7890 CICY manifolds (full dataset)
- **dataset pruning**: no product spaces, no “very far” outliers (reduction of 0.49%)
- $h^{1,1} \in [1, 16]$ and $h^{2,1} \in [15, 86]$
- 80% training, 10% validation, 10% test
- choose **regression**, but evaluate using **accuracy** (round the result)

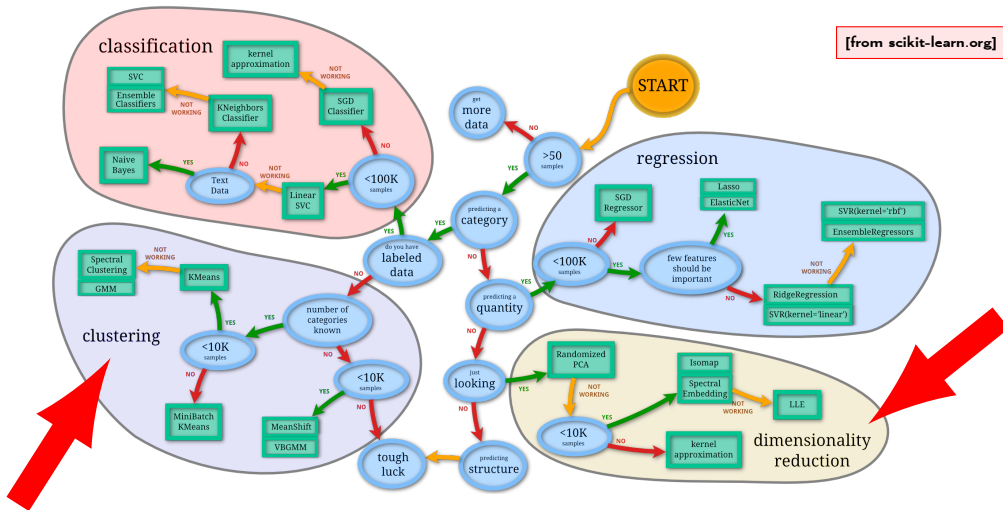


[from scikit-learn.org]



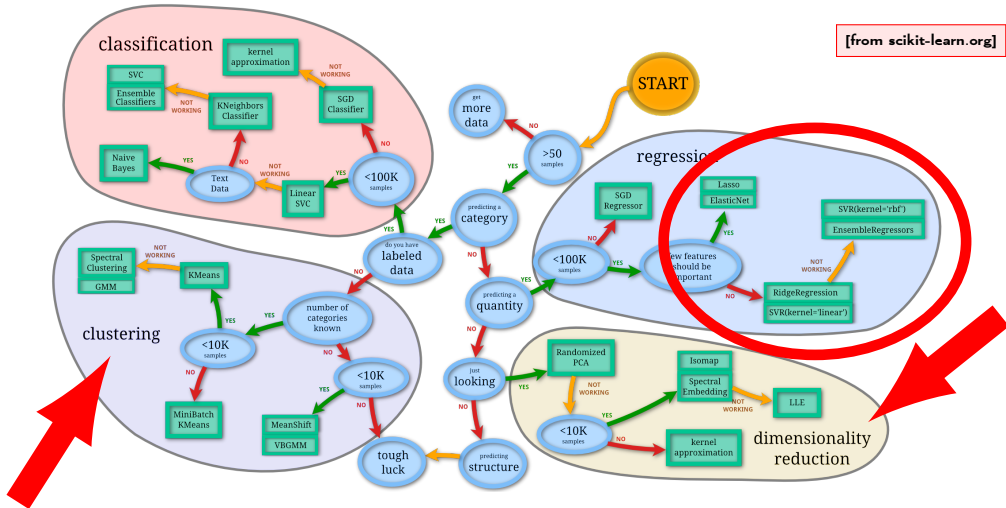
Machine Learning

[from scikit-learn.org]



Machine Learning

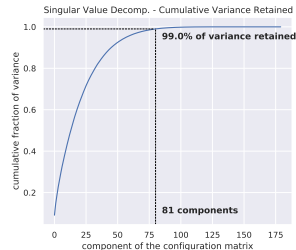
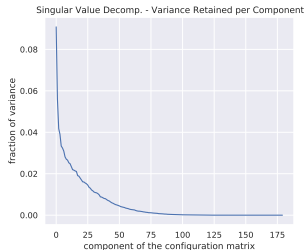
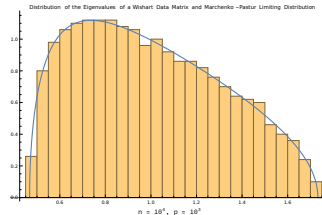
[from scikit-learn.org]



A Word on PCA

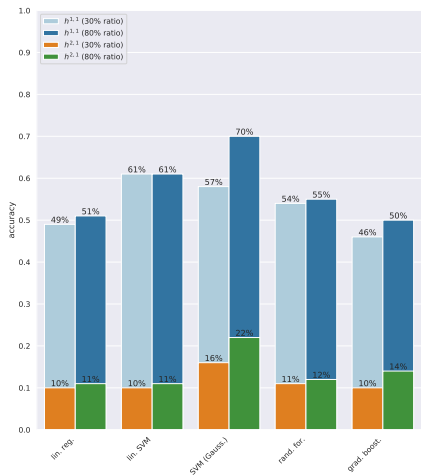
What is PCA for a $X \in \mathbb{R}^{n \times p}$?

- find new coordinates to “**put the variance in order**”
- equivalently compute the **eigenvectors** of XX^T or the **singular values** of X
- isolate **the signal** from the **background**
- ease the machine learning job of finding a better representation of the input



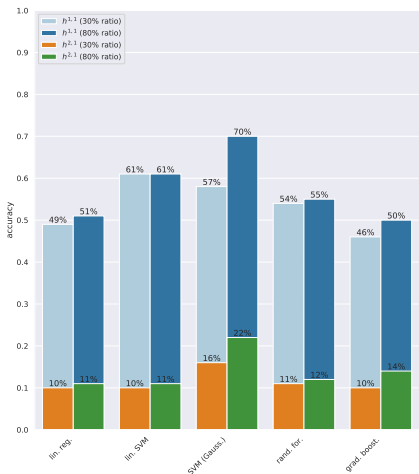
Machine Learning Results

Configuration Matrix Only

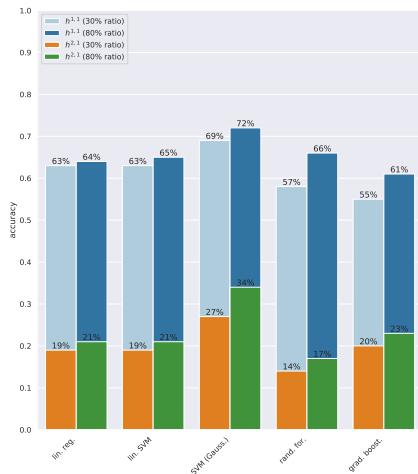


Machine Learning Results

Configuration Matrix Only



Best Training Set



Artificial Intelligence and Neural Networks

- use **gradient descent** to optimise **weights**
- learn highly **non linear** representations of the input
- can be “large” to have enough parameters
- can be “deep” to learn **complicated functions**

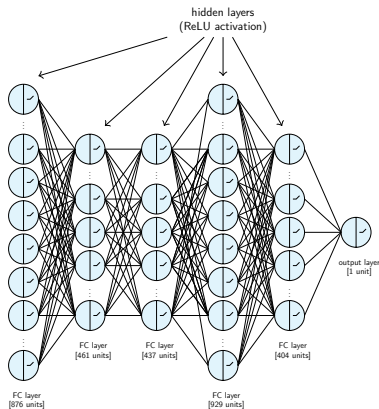
Neural Networks

fully connected: $a^{(i)\{l+1\}} = \phi(a^{(i)\{l\}} \cdot W^{\{l\}} + b^{\{l\}} \mathbf{1})$

convolutional: $a^{(i)\{l+1\}} = \phi(a^{(i)\{l\}} * W^{\{l\}} + b^{\{l\}} \mathbf{1})$

Non linearity ensured by:

$$\phi(z) = \text{ReLU}(z) = \max(0, z)$$

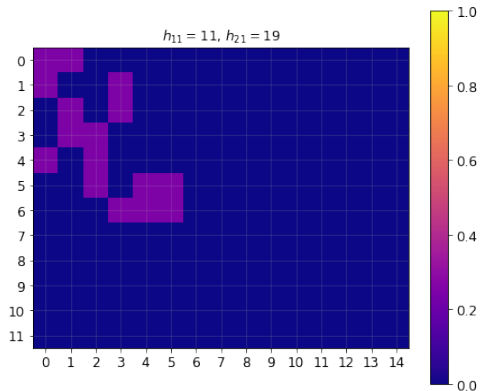


[rendition of the neural network in Bull et al. (2018)]

Convolutional Neural Networks

Why convolutional?

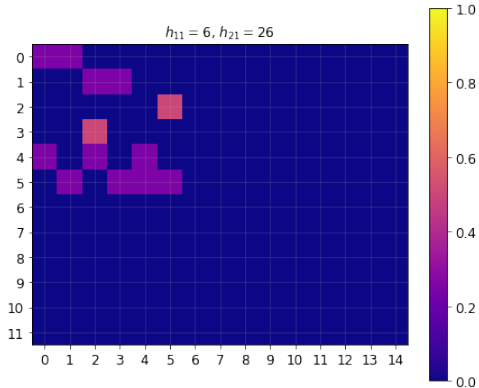
- retain **spacial** awareness



Convolutional Neural Networks

Why convolutional?

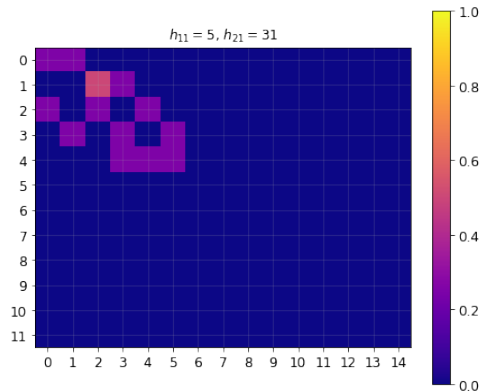
- retain **spacial awareness**
- smaller **no. of parameters**
($\approx 2 \times 10^5$ vs. $\approx 2 \times 10^6$)



Convolutional Neural Networks

Why convolutional?

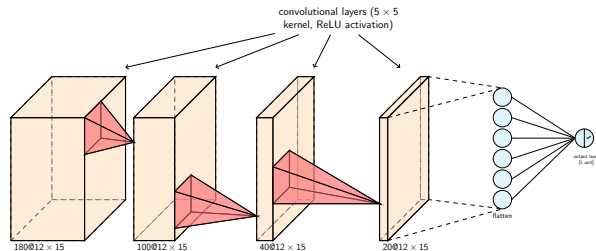
- retain **spacial awareness**
- smaller **no. of parameters**
($\approx 2 \times 10^5$ vs. $\approx 2 \times 10^6$)
- weights are **shared**



Convolutional Neural Networks

Why convolutional?

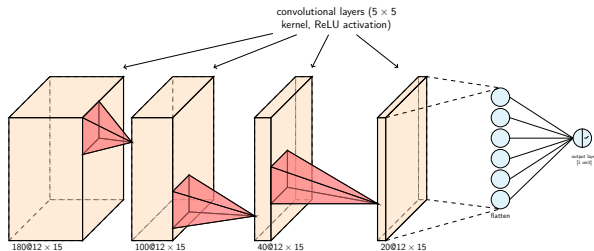
- retain **spacial awareness**
- smaller **no. of parameters**
($\approx 2 \times 10^5$ vs. $\approx 2 \times 10^6$)
- weights are **shared**
- CNN can isolate “**defining features**”



Convolutional Neural Networks

Why convolutional?

- retain **spacial awareness**
- smaller **no. of parameters**
($\approx 2 \times 10^5$ vs. $\approx 2 \times 10^6$)
- weights are **shared**
- CNN can isolate “**defining features**”
- find patterns as in **computer vision**



Inception Neural Networks

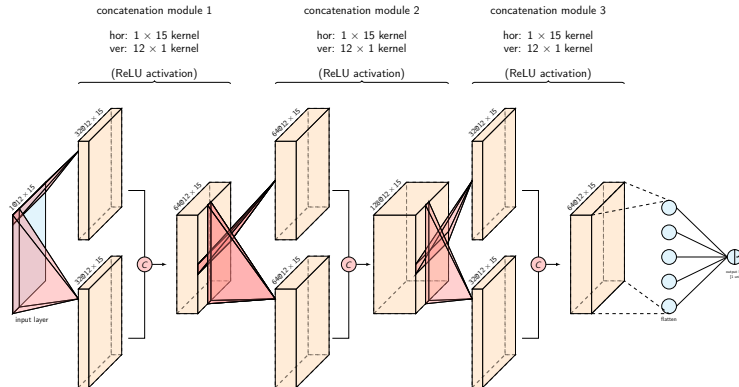
Recent development by Google's deep learning teams led to:

- neural networks with **better generalisation properties**
- smaller networks (both parameters and depth)
- different **concurrent kernels** (e.g. one over **equations** one over **coordinates**)

Inception Neural Networks

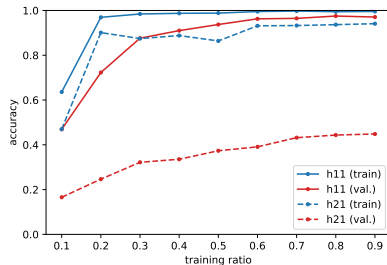
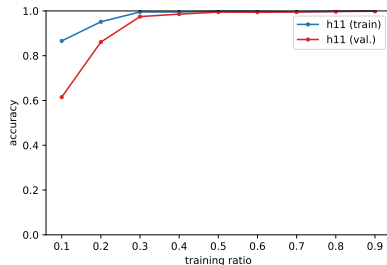
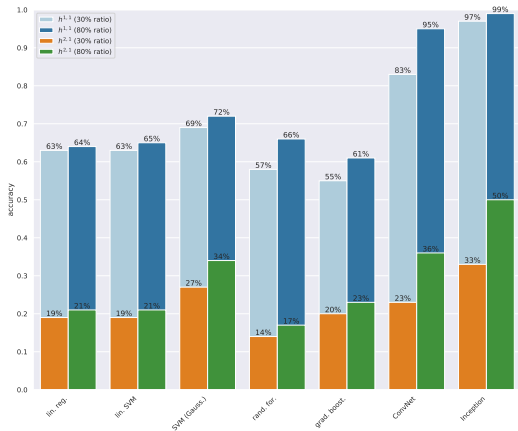
Recent development by Google's deep learning teams led to:

- neural networks with **better generalisation properties**
- smaller networks (both parameters and depth)
- different **concurrent kernels** (e.g. one over **equations** one over **coordinates**)



Deep Learning Results and Generalisation Properties

Best Training Set



A Few Comments and Future Directions

Why deep learning in physics?

- reliable **predictive method**

A Few Comments and Future Directions

Why deep learning in physics?

- reliable **predictive method** (provided good data analysis)

A Few Comments and Future Directions

Why deep learning in physics?

- reliable **predictive method** (provided good data analysis)
- reliable **source of inspiration**

A Few Comments and Future Directions

Why deep learning in physics?

- reliable **predictive method** (provided good data analysis)
- reliable **source of inspiration** (provided good data analysis)

A Few Comments and Future Directions

Why deep learning in physics?

- reliable **predictive method** (provided good data analysis)
- reliable **source of inspiration** (provided good data analysis)
- reliable **generalisation method**

A Few Comments and Future Directions

Why deep learning in physics?

- reliable **predictive method** (provided good data analysis)
- reliable **source of inspiration** (provided good data analysis)
- reliable **generalisation method** (provided good data analysis)

A Few Comments and Future Directions

Why deep learning in physics?

- reliable **predictive method** (provided good data analysis)
- reliable **source of inspiration** (provided good data analysis)
- reliable **generalisation method** (provided good data analysis)
- **CNNs are powerful tools** (this is the *first time in physics!*)

A Few Comments and Future Directions

Why deep learning in physics?

- reliable **predictive method** (provided good data analysis)
- reliable **source of inspiration** (provided good data analysis)
- reliable **generalisation method** (provided good data analysis)
- **CNNs are powerful tools** (this is the *first time in physics!*)
- interdisciplinary approach = win-win situation!

A Few Comments and Future Directions

Why deep learning in physics?

- reliable **predictive method** (provided good data analysis)
- reliable **source of inspiration** (provided good data analysis)
- reliable **generalisation method** (provided good data analysis)
- **CNNs are powerful tools** (this is the *first time in physics!*)
- interdisciplinary approach = win-win situation!

What now?

- representation learning $\Rightarrow$ what is the best way to represent CICYs?

A Few Comments and Future Directions

Why deep learning in physics?

- reliable **predictive method** (provided good data analysis)
- reliable **source of inspiration** (provided good data analysis)
- reliable **generalisation method** (provided good data analysis)
- **CNNs are powerful tools** (this is the *first time in physics!*)
- interdisciplinary approach = win-win situation!

What now?

- representation learning $\Rightarrow$ what is the best way to represent CICYs?
- study invariances $\Rightarrow$ invariances should not influence the result (graph representations?)

A Few Comments and Future Directions

Why deep learning in physics?

- reliable **predictive method** (provided good data analysis)
- reliable **source of inspiration** (provided good data analysis)
- reliable **generalisation method** (provided good data analysis)
- **CNNs are powerful tools** (this is the *first time in physics!*)
- interdisciplinary approach = win-win situation!

What now?

- representation learning $\Rightarrow$ what is the best way to represent CICYs?
- study invariances $\Rightarrow$ invariances should not influence the result (graph representations?)
- higher dimensions $\Rightarrow$ what about CICY 4-folds?

A Few Comments and Future Directions

Why deep learning in physics?

- reliable **predictive method** (provided good data analysis)
- reliable **source of inspiration** (provided good data analysis)
- reliable **generalisation method** (provided good data analysis)
- **CNNs are powerful tools** (this is the *first time in physics!*)
- interdisciplinary approach = win-win situation!

What now?

- representation learning $\Rightarrow$ what is the best way to represent CICYs?
- study invariances $\Rightarrow$ invariances should not influence the result (graph representations?)
- higher dimensions $\Rightarrow$ what about CICY 4-folds?
- geometric deep learning $\Rightarrow$ explain the geometry of the “AI” behind deep learning!

A Few Comments and Future Directions

Why deep learning in physics?

- reliable **predictive method** (provided good data analysis)
- reliable **source of inspiration** (provided good data analysis)
- reliable **generalisation method** (provided good data analysis)
- **CNNs are powerful tools** (this is the *first time in physics!*)
- interdisciplinary approach = win-win situation!

What now?

- representation learning $\Rightarrow$ what is the best way to represent CICYs?
- study invariances $\Rightarrow$ invariances should not influence the result (graph representations?)
- higher dimensions $\Rightarrow$ what about CICY 4-folds?
- geometric deep learning $\Rightarrow$ explain the geometry of the “AI” behind deep learning!
- reinforcement learning $\Rightarrow$ give the rules, not the result!

The End?



THANK YOU!