

D-branes and Deep Learning

Theoretical and Computational Aspects in String Theory

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Polyakov's Action

$$S_P[\gamma, X, \psi] = -\frac{1}{4\pi} \int_{-\infty}^{+\infty} d\tau \int_0^\ell d\sigma \sqrt{-\det \gamma} \gamma^{\alpha\beta} \left(\frac{2}{\alpha'} \partial_\alpha X^\mu \partial_\beta X^\nu + \psi^\mu \rho_\alpha \partial_\beta \psi^\nu \right) \eta_{\mu\nu}$$

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Symmetries:

- **Poincaré transf.** $X'^\mu = \Lambda^\mu_\nu X^\nu + c^\mu$
- **2D diff.** $\gamma'_{\alpha\beta} = (J^{-1})_{\alpha\beta}^{\lambda\rho} \gamma_{\lambda\rho}$
- **Weyl transf.** $\gamma'_{\alpha\beta} = e^{2\omega} \gamma_{\alpha\beta}$

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Conformal symmetry:

- **vanishing** stress-energy tensor: $\mathcal{T}_{\alpha\beta} = 0$
- **traceless** stress-energy tensor: $\text{tr } \mathcal{T} = 0$
- **conformal gauge** $\gamma_{\alpha\beta} = e^\Phi \eta_{\alpha\beta}$

Action Principle and Conformal Symmetry

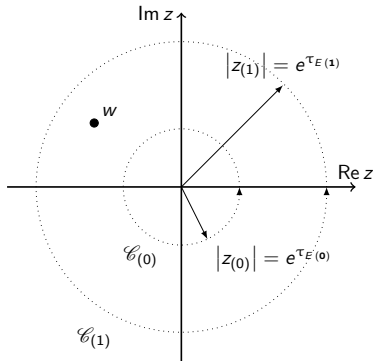
Let $z = e^{\tau_E + i\sigma} \Rightarrow \bar{\partial}\mathcal{T}(z) = \partial\bar{\mathcal{T}}(\bar{z}) = 0$:

$$\mathcal{T}(z) \Phi_h(w) \stackrel{z \rightarrow w}{\sim} \frac{h}{(z-w)^2} \Phi_h(w) + \frac{1}{z-w} \partial_w \Phi_h(w)$$

$$\mathcal{T}(z) \mathcal{T}(w) \stackrel{z \rightarrow w}{\sim} \frac{\frac{c}{2}}{(z-w)^4} + \mathcal{O}((z-w)^{-2})$$

Virasoro algebra $\mathcal{V} \oplus \bar{\mathcal{V}}$

$$\begin{aligned} \begin{bmatrix} L_n^{(-)} & \bar{L}_m^{(-)} \end{bmatrix} &= (n-m) L_{n+m}^{(-)} + \frac{c}{12} n(n^2-1) \delta_{n+m,0} \\ \begin{bmatrix} L_n & \bar{L}_m \end{bmatrix} &= 0 \end{aligned}$$



Action Principle and Conformal Symmetry

Superstrings in D dimensions:

$$\mathcal{T}(z) = -\frac{1}{\alpha'} \partial X(z) \cdot \partial X(z) - \frac{1}{2} \psi(z) \cdot \partial \psi(z) \quad \Rightarrow \quad c = \frac{3}{2}D$$

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$(\lambda, 0) / (1 - \lambda, 0)$ **Ghost System**

Introduce anti-commuting (b, c) and commuting (β, γ) conformal fields:

$$S_{\text{ghost}}[b, c, \beta, \gamma] = \frac{1}{2\pi} \iint dz d\bar{z} (b(z) \bar{\partial} c(z) + \beta(z) \bar{\partial} \gamma(z))$$

where $\lambda_b = 2$ and $\lambda_c = -1$, and $\lambda_\beta = \frac{3}{2}$ and $\lambda_\gamma = -\frac{1}{2}$.

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Consequence:

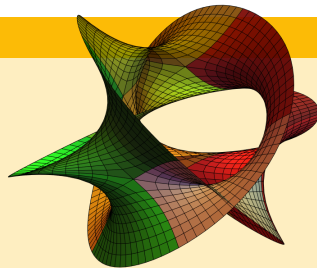
$$c_{\text{full}} = c + c_{\text{ghost}} = 0 \quad \Leftrightarrow \quad D = 10.$$

Extra Dimensions and Compactification

Compactification

$$\mathcal{M}^{1,9} = \mathcal{M}^{1,3} \otimes \mathcal{X}_6$$

- $\mathcal{X}_6$ is a **compact** manifold
- $N = 1$ **supersymmetry** is preserved in 4D
- algebra of $SU(3) \otimes SU(2) \otimes U(1)$ in arising **gauge group**

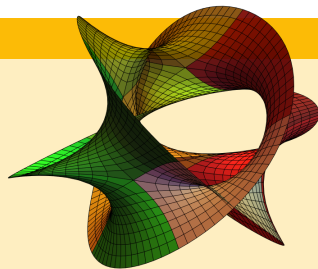


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Kähler manifolds (M, g) such that

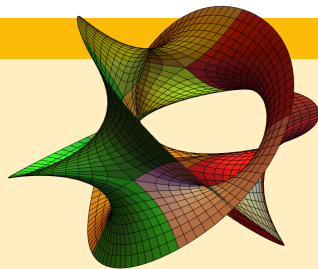
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Characterised by Hodge numbers

$$h^{r,s} = \dim_{\mathbb{C}} H_{\bar{\partial}}^{r,s}(M, \mathbb{C})$$

counting the no. of harmonic (r, s) -forms.

D-branes and Open Strings

Polyakov's action naturally introduces Neumann b.c.:

$$\partial_\sigma X(\tau, \sigma) \Big|_{\sigma=0}^{\sigma=\ell} = 0$$

satisfied by **open and closed** strings living in D dimensions s.t. $\square X = 0$.

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T-duality

$$X(z, \bar{z}) = X(z) + \overline{X}(\bar{z}) \quad \xrightarrow{T} \quad X(z) - \overline{X}(\bar{z}) = Y(z, \bar{z}) = Y(z) + \overline{Y}(\bar{z})$$

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Resulting effect (repeated $p \leq D - 1$ times) leads to **Dirichlet b.c.:**

$$\partial_\sigma X^i(\tau, \sigma) \Big|_{\sigma=0}^{\sigma=\ell} = 0 \xrightarrow{T} \partial_\tau Y^i(\tau, \sigma) \Big|_{\sigma=0}^{\sigma=\ell} = 0 \quad \forall i = 1, 2, \dots, p$$

thus **open strings** can be **constrained** to $D(D - p - 1)$ -branes.

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Introducing Dp -branes breaks $\text{ISO}(1, D - 1) \rightarrow \text{ISO}(1, p) \otimes \text{SO}(D - 1 - p)$.

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Massless Spectrum (*irrep* of little group $\text{SO}(D-2)$)

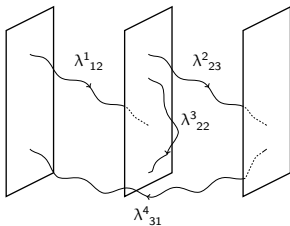
$$\mathcal{A}^\mu \leftrightarrow \alpha_{-1}^\mu |0\rangle \longrightarrow \begin{array}{ll} \mathcal{A}^A \leftrightarrow \alpha_{-1}^A |0\rangle, & A = 0, 1, \dots, p \\ \mathcal{A}^a \leftrightarrow \alpha_{-1}^a |0\rangle, & a = 1, 2, \dots, D-p-1 \end{array}$$

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Introducing **Chan-Paton factors** λ^r_{ij} , when branes are **coincident**:

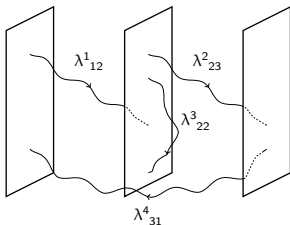
$$\bigoplus_{r=1}^N \text{U}_r(1) \longrightarrow \text{U}(N)$$

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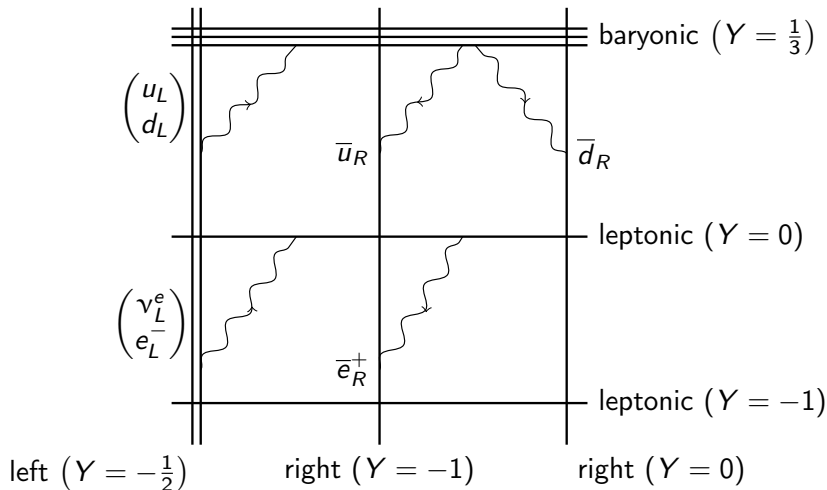


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Build gauge bosons, fermions and scalars.

Standard Model-like Scenarios



Intersecting D-branes

Consider N intersecting $D6$ -branes filling $\mathcal{M}^{1,3}$ and **embedded in** $\mathbb{R}^6$

Twist Fields Correlators

$$\left\langle \prod_{t=1}^{N_B} \sigma_{M_{(t)}}(x_{(t)}) \right\rangle = \mathcal{N} \left(\{x_{(t)}, M_{(t)}\}_{1 \leq t \leq N_B} \right) e^{-S_E(\text{cl}) \left(\{x_{(t)}, M_{(t)}\}_{1 \leq t \leq N_B} \right)}$$

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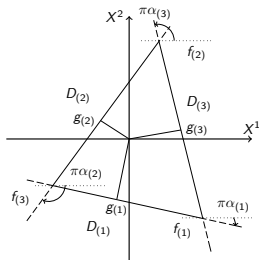
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D-branes in **factorised** internal space:

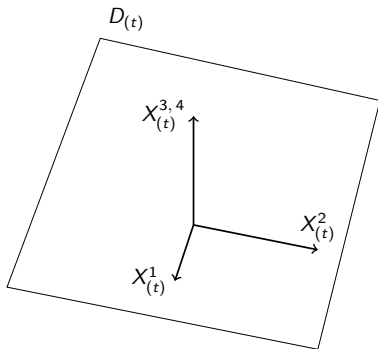
- **embedded as lines** in $\mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2$
- **relative rotations** are $\text{SO}(2) \simeq \text{U}(1)$ elements
- $S_{E(\text{cl})} \left(\{x_{(t)}, M_{(t)}\}_{1 \leq t \leq N_B} \right) \sim \text{Area} \left(\{f_{(t)}, R_{(t)}\}_{1 \leq t \leq N_B} \right)$

$SO(4)$ Rotations

Consider $\mathbb{R}^4 \times \mathbb{R}^2$ (focus on $\mathbb{R}^4$):

SO(4) Rotations

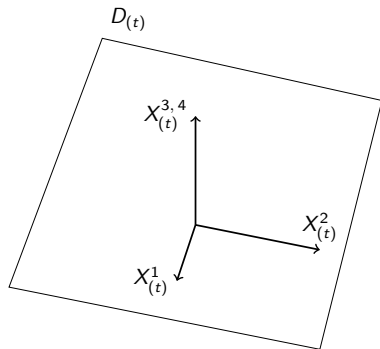
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$$(X_{(t)})^I = (R_{(t)})^I_J X^J - g_{(t)}^I$$

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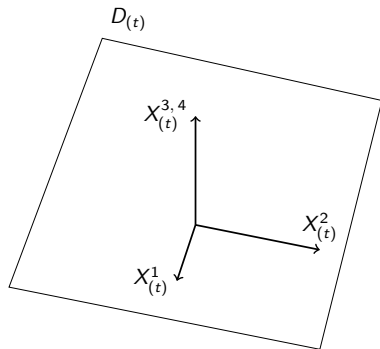
where

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that is

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$$[R_{(t)}] = \{R_{(t)} \sim \mathcal{O}_{(t)} R_{(t)}\}$$

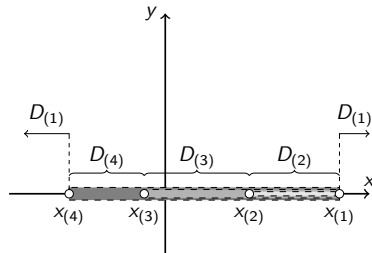
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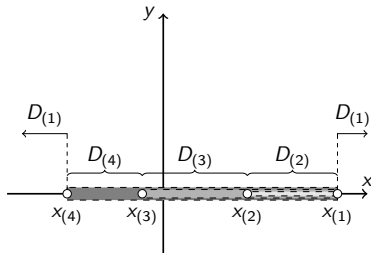
- consider $u = x + iy = e^{\tau_e + i\sigma}$ and $\bar{u} = u^*$
- let $x_{(t)} < x_{(t-1)}$ be the **worldsheet intersection points on real axis**
- $X_{(t)}^{1,2}$ are **Neumann**, $X_{(t)}^{3,4}$ are **Dirichlet**



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Branch Cuts and Discontinuities for $x \in D_{(t)}$

$$\begin{cases} \partial_u X(x + i0^+) = U_{(t)} \cdot \partial_{\bar{u}} \bar{X}(x - i0^+) = \left[R_{(t)}^{-1} \cdot (\sigma_3 \otimes \mathbb{1}_2) \cdot R_{(t)} \right] \cdot \partial_{\bar{u}} \bar{X}(x - i0^+) \\ X(x_{(t)}, x_{(t)}) = f_{(t)} \end{cases}$$

Doubling Trick and Spinor Representation

Doubling Trick

$$\partial_z \mathcal{X}(z) = \begin{cases} \partial_u X(u) & \text{if } z \in \mathcal{H}_{>}^{(\bar{t})} \\ U_{(\bar{t})} \partial_{\bar{u}} \bar{X}(\bar{u}) & \text{if } z \in \mathcal{H}_{<}^{(\bar{t})} \end{cases} \Rightarrow \begin{aligned} \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i} \delta_+) &= \mathcal{U}_{(t, t+1)} \partial_z \mathcal{X}(x_{(t)} + \delta_+), \\ \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i} \delta_-) &= \tilde{\mathcal{U}}_{(t, t+1)} \partial_z \mathcal{X}(x_{(t)} + \delta_-), \end{aligned}$$

where $\mathcal{H}_{\gtrless}^{(t)} = \{z \in \mathbb{C} \mid \text{Im } z \gtrless 0 \text{ or } z \in D_{(t)}\}$ and $\delta_{\pm} = \eta \pm i0^+$.

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Use Pauli matrices $\tau = (i \mathbb{1}_2, \vec{\sigma})$:

$$\partial_z \mathcal{X}_{(s)}(z) = \partial_z \mathcal{X}'(z) \tau_I \Rightarrow \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i} \delta_{\pm}) = \overset{(\sim)}{\mathcal{L}}_{(t, t+1)} \partial_z \mathcal{X}(x_{(t)} + \delta_{\pm}) \overset{(\sim)}{\mathcal{R}}_{(t, t+1)}$$

where

$$\overset{(\sim)}{\mathcal{L}}_{(t, t+1)} \in \text{SU}(2)_L \quad \text{and} \quad \overset{(\sim)}{\mathcal{R}}_{(t, t+1)} \in \text{SU}(2)_R$$



BBB

b

CCC

C