D-branes and Deep Learning

Theoretical and Computational Aspects in String Theory

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Polyakov's Action

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Symmetries:	
Poincaré transf.:	$X'^\mu = \Lambda^\mu{}_\nu X^\nu + c^\mu$
2D diff.:	$\gamma'_{\alpha\beta} = \left(\mathrm{J}^{-1}\right)_{\alpha\beta}{}^{\lambda\rho} \gamma_{\lambda\rho}$
Weyl transf.:	$\gamma'_{\alpha\beta} = e^{2\omega} \gamma_{\alpha\beta}$

Conformal symmetry:	
vanishing stress-energy tensor:	$\mathcal{T}_{\alpha\beta} = 0$
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conformal gauge:	$\gamma_{\alpha\beta} = e^{\phi} \eta_{\alpha\beta}$

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Conformal properties fixed by **OPE**s.

Action Principle and Conformal Symmetry

Superstrings in D dimensions $\longrightarrow$ *Virasoro algebra* (central extension of de Witt's algebra):

$$\mathcal{T}(z) = -\frac{1}{\alpha'}\partial X(z) \cdot \partial X(z) - \frac{1}{2}\psi(z) \cdot \partial\psi(z) \quad \Rightarrow \quad c = \frac{3}{2}D$$

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$(\lambda, 0)$ / $(1 - \lambda, 0)$ Ghost System

Introduce anti-commuting $(b,\ c)$ and commuting $(\beta,\ \gamma)$ conformal fields:

$$S_{\text{ghost}}[b,\ c,\ \beta,\ \gamma] = \frac{1}{2\pi} \iint \mathrm{d}z\, \mathrm{d}\bar{z}\, \left(b(z)\, \bar{\partial} c(z) + \beta(z)\, \bar{\partial}\gamma(z) \right)$$

where $\lambda_b = 2$ and $\lambda_c = -1$, and $\lambda_\beta = \frac{3}{2}$ and $\lambda_\gamma = -\frac{1}{2}$.

[Friedan, Martinec, Shenker (1986)]

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Consequence:

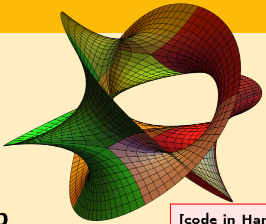
$$c_{\text{full}} = c + c_{\text{ghost}} = 0 \quad \Leftrightarrow \quad D = 10.$$

Extra Dimensions and Compactification

Compactification

$$\mathscr{M}^{1,9} = \mathscr{M}^{1,3} \otimes \mathscr{X}_6$$

- $\mathscr{X}_6$ is a **compact** manifold
- $N = 1$ **supersymmetry** preserved in 4D
- algebra of $\mathrm{SU}(3) \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1)$ in arising **gauge group**



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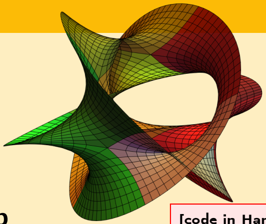

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Kähler manifolds (M, g) such that:

- $\dim_{\mathbb{C}} M = m$
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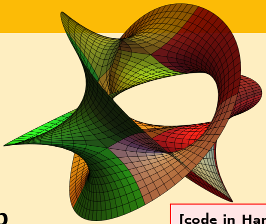

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Characterised by **Hodge numbers**

$$h^{r,s} = \dim_{\mathbb{C}} H^{r,s}_{\bar{\partial}}(M, \mathbb{C})$$

counting the no. of harmonic (r, s)-forms.

D-branes and Open Strings

Polyakov's action naturally introduces **Neumann b.c.** for **open strings**:

$$\partial_\sigma X(\tau,\sigma)\Big|_{\sigma=0}^{\sigma=\ell} = 0$$

satisfied by **open and closed strings** in D dim. s.t. $\Box X = 0 \Rightarrow X(z,\bar{z}) = X(z) + \overline{X}(\bar{z})$.

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T-duality

Dirichlet b.c. consequence of **T-duality** on p directions:

$$\overline{X}(z) \mapsto -\overline{X}(z) \quad \Rightarrow \quad \partial_\sigma X^i(\tau,\sigma)\Big|_{\sigma=0}^{\sigma=\ell} = 0 \quad \xrightarrow{T-duality} \quad \partial_\tau \widetilde{X}^i(\tau,\sigma)\Big|_{\sigma=0}^{\sigma=\ell} = 0$$

thus **open strings** can be **constrained** to $D(D-p-1)$-branes.

[Polchinski (1995, 1996)]

D-branes and Open Strings

Introducing Dp-branes breaks $\boxed{\mathrm{ISO}(1,\, D-1) \to \mathrm{ISO}(1,p) \otimes \mathrm{SO}(D-1-p)}$:

$$\mathcal{A}^{\mu} \to \left(\mathcal{A}^{A},\, \mathcal{A}^{a}\right) \quad \Rightarrow \quad \mathrm{U}(1) \text{ theory in } p+1 \text{ dimensions (and scalars)}$$

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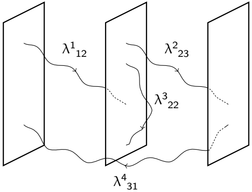

[Chan, Paton (1969)]

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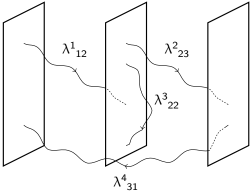

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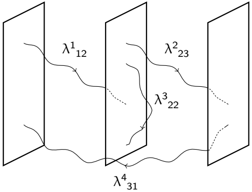

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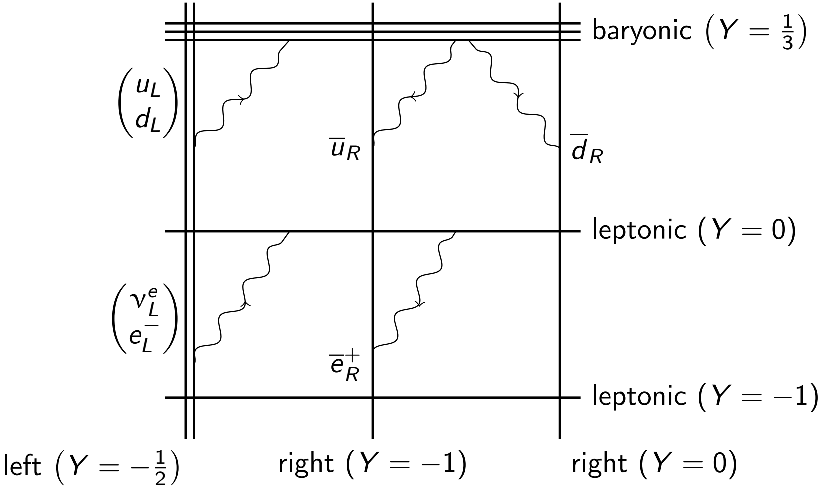
Build gauge bosons, fermions and scalars.

Standard Model-like Scenarios



[Zwiebach (2009)]

Intersecting D-branes

Consider N intersecting $D6$-branes filling $\mathscr{M}^{1,3}$ and **embedded in** $\mathbb{R}^6$

Twist Fields Correlators

$$\left\langle \prod_{t=1}^{N_B} \sigma_{\mathrm{M}_{(t)}}\left(x_{(t)}\right) \right\rangle = \mathcal{N}\left(\left\{x_{(t)},\, \mathrm{M}_{(t)}\right\}_{1 \le t \le N_B}\right) e^{-S_{E\,(\mathbf{cl})}\left(\left\{x_{(t)},\, \mathrm{M}_{(t)}\right\}_{1 \le t \le N_B}\right)}$$

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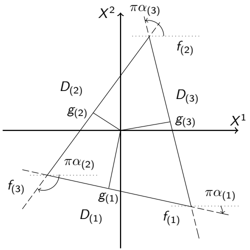

D-branes in **factorised** internal space:

- **embedded as lines** in $\mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2$
- **relative rotations** are $\mathrm{SO}(2) \simeq \mathrm{U}(1)$ elements
- $S_{E\,(\mathrm{cl})}\left(\left\{x_{(t)},\, \mathrm{M}_{(t)}\right\}_{1\le t\le N_B}\right) \sim \mathrm{Area}\left(\left\{f_{(t)},\, \mathrm{R}_{(t)}\right\}_{1\le t\le N_B}\right)$

[Cremades, Ibanez, Marchesano (2003); Pesando (2012)]

$\mathrm{SO}(4)$ Rotations

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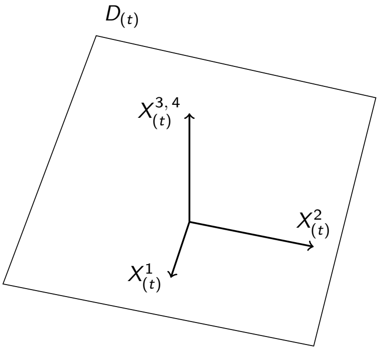

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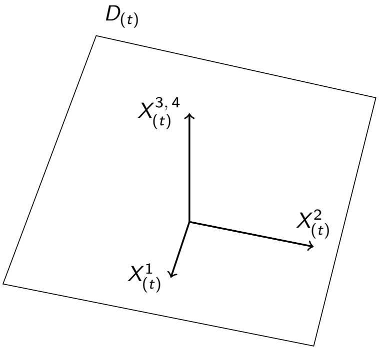

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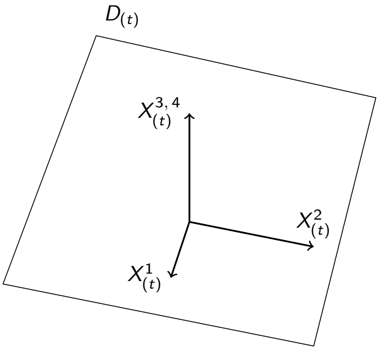

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that is

$$\left[R_{(t)}\right] = \left\{R_{(t)} \sim \mathcal{O}_{(t)} R_{(t)}\right\}$$

Boundary Conditions

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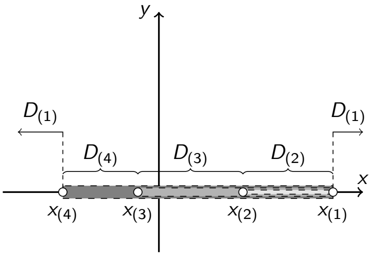
- $u = x + iy = e^{\tau_e + i\sigma}$ and $\overline{u} = u^*$
- $x_{(t)} < x_{(t-1)}$ **worldsheet intersection points**
- $X^{1,2}_{(t)}$ are **Neumann**, $X^{3,4}_{(t)}$ are **Dirichlet**



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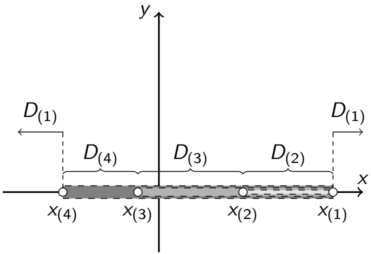

Branch Cuts and Discontinuities for $x \in D_{(t)}$

$$\begin{cases} \partial_u X(x + i0^+) & = U_{(t)} \cdot \partial_{\overline{u}} \overline{X}(x - i0^+) = \left[R^{-1}_{(t)} \cdot (\sigma_3 \otimes \mathbb{1}_2) \cdot R_{(t)} \right] \cdot \partial_{\overline{u}} \overline{X}(x - i0^+) \\ X(x_{(t)}, x_{(t)}) & = f_{(t)} \end{cases}$$

Doubling Trick and Spinor Representation

Doubling Trick

$$\partial_z \mathcal{X}(z) = \begin{cases} \partial_u X(u) & \text{if } z \in \mathscr{H}_{>}^{(\bar t)} \\ U_{(\bar t)}\, \partial_{\bar u} \overline{X}(\bar u) & \text{if } z \in \mathscr{H}_{<}^{(\bar t)} \end{cases} \quad \Rightarrow \quad \begin{aligned} \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i}\delta_+) &= \mathcal{U}_{(t,\, t+1)}\, \partial_z \mathcal{X}(x_{(t)} + \delta_+), \\ \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i}\delta_-) &= \widetilde{\mathcal{U}}_{(t,\, t+1)}\, \partial_z \mathcal{X}(x_{(t)} + \delta_-), \end{aligned}$$

where $\mathscr{H}_{\gtrless}^{(t)} = \left\{ z \in \mathbb{C} \mid \operatorname{Im} z \gtrless 0 \text{ or } z \in D_{(t)} \right\}$ and $\delta_\pm = \eta \pm i0^+$.

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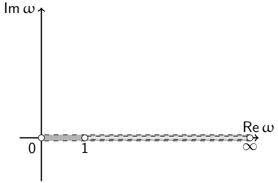
Use Pauli matrices $\tau = (i\, \mathbb{1}_2, \vec{\sigma})$:

$$\partial_z \mathcal{X}_{(s)}(z) = \partial_z \mathcal{X}^I(z)\, \tau_I \quad \Rightarrow \quad \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i}\, \delta_\pm) = \overset{(\sim)}{\mathcal{L}}_{(t,\,t+1)}\, \partial_z \mathcal{X}(x_{(t)} + \delta_\pm)\, \overset{(\sim)}{\mathcal{R}}_{(t,\,t+1)}$$

where

$$\overset{(\sim)}{\mathcal{L}}_{(t,\,t+1)} \in \mathrm{SU}(2)_L \quad \text{and} \quad \overset{(\sim)}{\mathcal{R}}_{(t,\,t+1)} \in \mathrm{SU}(2)_R$$

Hypergeometric Basis



Sum over all contributions:

$$\partial_z \mathcal{X}(z) = \sum_{l,\, r=-\infty}^{+\infty} c_{lr} \left(-\omega_z\right)^{A_{lr}} \left(1-\omega_z\right)^{B_{lr}} B_{0,\, l}^{(L)}(\omega_z) \left(B_{0,\, r}^{(R)}(\omega_z)\right)^T$$

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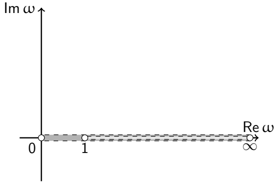

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Basis of Solutions

$$B_{0,\,n}(\omega_z) = \begin{pmatrix} 1 & 0 \\ 0 & K_n \end{pmatrix} \begin{pmatrix} \frac{1}{\Gamma(c_n)}\, {}_2F_1\left(a_n,\, b_n;\, c_n;\, \omega_z\right) \\ \frac{(-\omega_z)^{1-c_n}}{\Gamma(2-c_n)}\, {}_2F_1\left(a_n+1-c_n,\, b_n+1-c_n;\, 2-c_n;\, \omega_z\right) \end{pmatrix}$$

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Sequence of the operations:

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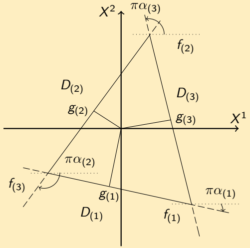
Physical Interpretation



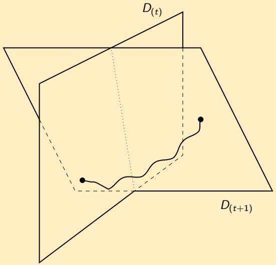
$$
\begin{aligned}
2\pi\alpha' \, S_{\mathbb{R}^4}\Big|_{\text{on-shell}} &= \sum_{t=1}^{3} \left(\frac{1}{2} \left| g_{(t)}^{\perp} \right| \left| f_{(t-1)} - f_{(t)} \right| \right) \\
&= \text{Area}\left(\left\{ f_{(t)} \right\}_{1 \le t \le N_B} \right)
\end{aligned}
$$

The Solution

Sequence of the operations:

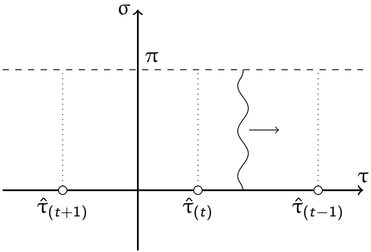
1. rotation matrix = monodromy matrix
2. contiguity relations ⇒ independent hypergeometrics
3. finite action ⇒ 2 solutions (no. of d.o.f. is correctly saturated)
4. boundary conditions ⇒ fix free constants c_{lr}

Physical Interpretation



- strings no longer confined to plane
- strings form a *small bump* from the D-brane
- classical action **larger** than factorised case

Fermions on the Strip



Action of Boundary Changing Operators

$$\begin{cases} \psi^i_-(\tau, 0) & = \left(R_{(t)}\right)^I{}_J \, \psi^J_+(\tau, 0) \quad \text{for } \tau \in \left(\hat{\tau}_{(t)}, \hat{\tau}_{(t-1)}\right) \\ \psi^I_-(\tau, \pi) & = -\psi^I_+(\tau, \pi) \quad \text{for } \tau \in \mathbb{R} \end{cases}$$

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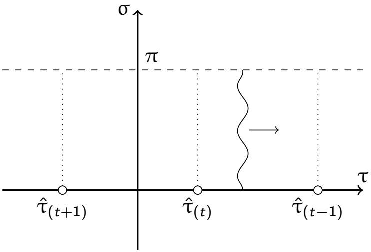

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Stress-energy Tensor

$$\mathcal{T}_{\pm\pm}(\xi_\pm) = -i\,\frac{T}{4}\,\psi^*_{\pm,I}(\xi_\pm)\,\overleftrightarrow{\partial}\psi^I_\pm(\xi_\pm) \quad \Rightarrow \quad \begin{cases} \dot{\mathrm{H}}(\tau) &= 0 \quad \Leftrightarrow \quad \tau \in \left(\tau_{(t)},\, \tau_{(t-1)}\right) \\ \dot{\mathrm{P}}(\tau) &\neq 0 \end{cases}$$

Conserved Product and Operators

Expand on a **basis of solutions**

$$\psi_{\pm}(\xi_{\pm}) = \sum_{n=-\infty}^{+\infty} b_n \, \psi_n(\xi_{\pm}) \qquad \Rightarrow \qquad \Psi(z) = \begin{cases} \psi_{E,+}(u) & \text{if } z \in \mathscr{H}_{>}^{(\bar{t})} \\ \psi_{E,-}(u) & \text{if } z \in \mathscr{H}_{<}^{(\bar{t})} \end{cases}$$

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Conserved Product and Dual Basis

$$\langle\!\langle {}^*\psi_n, \psi_m \rangle = 2\pi\mathcal{N} \oint \frac{\mathrm{d}z}{2\pi i} {}^*\Psi_n{}^* \Psi_m = \delta_{n,m} \qquad \Rightarrow \qquad \left\langle\!\left\langle {}^*\Psi_n{}^{(*)}, \Psi^{(*)} \right\rangle\right. = b_n^{(\dagger)}$$

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Derive the **algebra of operators:**

$$\left[b_n, \, b_m^\dagger \right]_+ = \frac{2\mathcal{N}}{T} \, \langle\!\langle {}^*\Psi_n{}^*, \, \Psi_m^* \rangle$$

Twisted Complex Fermions

Consider the case $R_{(t)} = e^{i\pi\alpha_{(t)}} \in \mathrm{U}(1)$:

$$\Psi(x_{(t)} + e^{2\pi i}\delta) = e^{i\pi\epsilon_{(t)}}\,\Psi(x_{(t)} + \delta)$$

where

$$\epsilon_{(t)} = \alpha_{(t+1)} - \alpha_{(t)} + \theta\left(\alpha_{(t)} - \alpha_{(t+1)} - 1\right) - \theta\left(\alpha_{(t+1)} - \alpha_{(t)} - 1\right)$$

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Basis of Solutions

$$\Psi_n\big(z;\,\{x_{(t)}\}\big) = \mathcal{N}_\Psi\, z^{-n} \prod_{t=1}^{N} \left(1 - \frac{z}{x_{(t)}}\right)^{n_{(t)} + \frac{\epsilon_{(t)}}{2}}$$

$$^{*}\Psi_n\big(z;\,\{x_{(t)}\}\big) = \frac{1}{2\pi\mathcal{N}\mathcal{N}_\Psi}\, z^{n-1} \prod_{t=1}^{N} \left(1 - \frac{z}{x_{(t)}}\right)^{-\widetilde{n}_{(t)} + \frac{\epsilon_{(t)}}{2}}$$

Vacua

Define the **vacuum** with respect to b_n:

$$b_n \left| \left\{ x_{(t)} \right\} \right\rangle = 0 \quad \text{for} \quad n \geq 1$$

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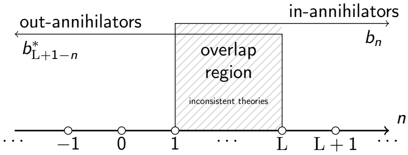
Theories are subject to **consistency conditions:**

$$\mathrm{L} = n_{(t)} + \widetilde{n}_{(t)}$$



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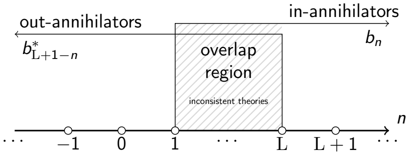

Stress-energy Tensor and CFT Approach

Compute the OPEs leading to the **stress-energy tensor:**

$$\mathcal{T}(z) = \frac{\pi T}{2} \mathcal{N}_{\Psi}^2 \sum_{n,\, m=-\infty}^{+\infty} : b_n\, b_m^* : \; z^{-n-m} \left[\frac{m-n}{2} + 2 \sum_{t=1}^{N} \frac{n_{(t)} + \frac{\epsilon_{(t)}}{2}}{z - x_{(t)}} \right] + \frac{1}{2} \left(\sum_{t=1}^{N} \frac{n_{(t)} + \frac{\epsilon_{(t)}}{2}}{z - x_{(t)}} \right)^2$$

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Invariant Vacuum and Spin Fields

$$\left| \left\{ x_{(t)} \right\} \right\rangle = \mathcal{N}\left(\left\{ x_{(t)} \right\} \right) \mathrm{R} \left[\prod_{t=1}^{M} S_{(t)}(x_{(t)}) \right] \left| 0 \right\rangle_{\mathrm{SL}_{\mathbf{2}}(\mathbb{R})}$$

Spin Fields Amplitudes

Equivalence with Bosonization

$$\partial_{x_{(t)}} \left\langle \left\{ x_{(t)} \right\} \middle| \left\{ x_{(t)} \right\} \right\rangle = \oint\limits_{x_{(t)}} \frac{\mathrm{d}z}{2\pi i} \frac{\left\langle \left\{ x_{(t)} \right\} \middle| \mathcal{T}(z) \middle| \left\{ x_{(t)} \right\} \right\rangle}{\left\langle \left\{ x_{(t)} \right\} \middle| \left\{ x_{(t)} \right\} \right\rangle}$$

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- alternative framework for amplitudes (extension to (non) Abelian twist/spin fields?)

Contents

A Few Words on a Theory of Everything

string theory = theory of everything = nuclear forces + gravity

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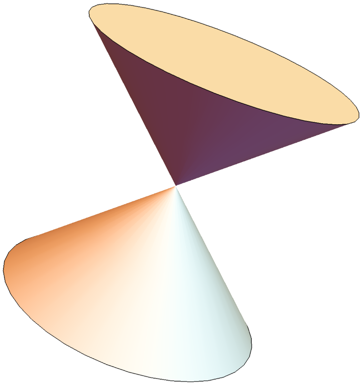

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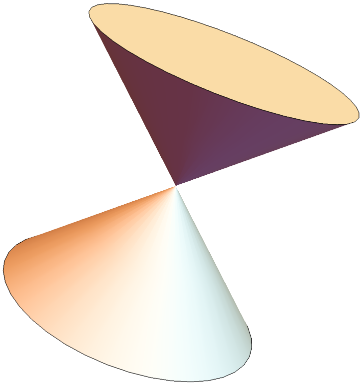

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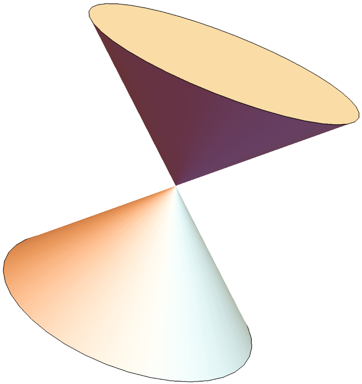

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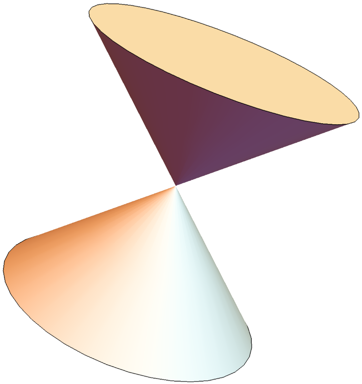

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$\Downarrow$

time-dependent orbifold models

[Craps, Kutasov, Rajesh (2002); Liu, Moore, Seiberg (2002)]

Orbifolds

Mathematics

- manifold M
- (Lie) group G
- *stabilizer* $G_p = \{g \in G \mid gp = p \in M\}$
- *orbit* $Gp = \{gp \in M \mid g \in G\}$
- charts $\phi = \pi \circ \mathscr{P}$ where:
  - $\mathscr{P} \colon U \subset \mathbb{R}^n \to U/G$
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- global orbit space M/G
- G group of isometries
- fixed points
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- singular limits of CY manifolds

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Use **time-dependent orbifolds** to model singularities in time

Cosmological Singularities

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divergent $\boxed{\text{closed string}}$ aplitudes $\Rightarrow$ gravitational backreaction?

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Divergences

Even in simple models (e.g. NBO, more on this later) the 4 tachyons amplitude is divergent **at tree level**:

$$A_4 \sim \int\limits_{q \sim \infty} \frac{\mathrm{d}q}{|q|} \mathscr{A}(q)$$

where

$$\mathscr{A}_{\text{closed}}(q) \sim q^{4-\alpha' \|\vec{p}_\perp\|^2} \qquad \text{and} \qquad \mathscr{A}_{\text{open}}(q) \sim q^{1-\alpha' \|\vec{p}_\perp\|^2} \operatorname{tr}([T_1,\ T_2]_+ [T_3,\ T_4]_+)$$

Null Boost Orbifold

Start from $\left(x^+,\, x^-,\, x^2,\, \vec{x}\right) \in \mathscr{M}^{1,\, D-1}$:

$$\begin{cases} u &= x^- \\ z &= \frac{x^2}{\Delta\, x^-} \\ v &= x^+ - \frac{1}{2}\frac{\left(x^2\right)^2}{x^-} \end{cases} \quad\Rightarrow\quad \mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}v + \left(\Delta\, u\right)^2\,\mathrm{d}z^2 + \delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j$$

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$$\kappa = -i(2\pi\Delta)J_{+2} = 2\pi\partial_z \Rightarrow z \sim z + 2\pi n$$

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Scalars on NBO:

$$\phi_{\{k_+,\, l,\, \vec{k},\, r\}}(u,\, v,\, z,\, \vec{x}) = e^{i\left(k_+ v + lz + \vec{k}\cdot\vec{x}\right)}\, \widetilde{\phi}_{\{k_+,\, l,\, \vec{k},\, r\}}(u) = \frac{e^{i\left(k_+ v + lz + \vec{k}\cdot\vec{x}\right)}}{\sqrt{(2\pi)^D\, \left|2\Delta k_+ u\right|}}\, e^{-i \frac{l^2}{2\Delta^2 k_+} \frac{1}{u} + i \frac{\left\|\vec{k}\right\|^2 + r}{2k_+} u}$$

Scalar QED Interactions

Scalar–photon interactions:

$$S_{\mathrm{sQED}}^{(\mathrm{int})} = \int_{\Omega} \mathrm{d}^D x \sqrt{-g} \left(-i\, e\, g^{\alpha\beta} a_\alpha (\phi^* \partial_\beta \phi - \partial_\beta \phi^* \phi) + e^2 g^{\alpha\beta} a_\alpha a_\beta |\phi|^2 - \frac{g_4}{4} |\phi|^4 \right)$$

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Terms involved:

$$\mathcal{I}^{[\nu]}_{\{N\}} = \int\limits_{-\infty}^{+\infty} \mathrm{d}u\, |\Delta\, u| u^\nu \prod_{i=1}^{N} \widetilde{\phi}_{\left\{ k_{+\,(i)},\, l_{(i)},\, \vec{k}_{(i)},\, r_{(i)} \right\}}(u)$$

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*most terms **do not converge** due to **isolated zeros** ($l_{(*)} \equiv 0$) and cannot be recovered even with a **distributional interpretions** due to the term $\propto u^{-1}$ in the exponentatial*

String and Field Theory

So far:

- field theory presents **divergences** (even sQED $\to$ eikonal?)

String and Field Theory

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- field theory presents **divergences** (even sQED $\to$ eikonal?)
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String and Field Theory

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- field theory presents **divergences** (even sQED $\to$ eikonal?)
- divergences are **not (only) gravitational**
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Massive String States

$$V_M(x;\, k,\, S,\, \xi) =: \left(\frac{i}{\sqrt{2\alpha'}}\, \xi \cdot \partial_x^2 X(x,\, x) + \left(\frac{i}{\sqrt{2\alpha'}} \right)^2 S_{\alpha\beta} \partial_x X^\alpha(x,\, x) \partial_x X^\beta(x,\, x) \right) e^{ik \cdot X(x,\, x)} :$$

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*string theory cannot do **better than field theory** (EFT) if the latter **does not exist** (even a Wilson line around z does not prevent such behaviour)*

Resolution and Motivation

Introduce the generalised NBO:

$$\begin{cases} u &= x^- \\ z &= \frac{1}{2x^-}\left(\frac{x^2}{\Delta_2} + \frac{x^3}{\Delta_3}\right) \\ w &= \frac{1}{2x^-}\left(\frac{x^2}{\Delta_2} - \frac{x^3}{\Delta_3}\right) \\ v &= x^+ - \frac{1}{2x^-}\left(\left(x^2\right)^2 + \left(x^3\right)^2\right) \end{cases} \quad \Rightarrow \quad \kappa = -2\pi i(\Delta_2 J_{+2} + \Delta_3 J_{+3}) = 2\pi\partial_z$$

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Distributional Interpretation

$$\widetilde{\phi}_{\{k_+,\, p,\, l,\, \vec{k},\, r\}}(u) = \frac{1}{2\sqrt{(2\pi)^D |\Delta_2 \Delta_3 k_+|}} \frac{1}{|u|} e^{-i\left(\frac{1}{8k_+ u}\left[\frac{(l+p)^2}{\Delta_2^2} + \frac{(l-p)^2}{\Delta_3^2}\right] - \frac{\|\vec{k}\|^2 + r}{2k_+} u\right)}$$

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*spacetime singularities are **hidden into contact terms** and interactions with **massive states** (the gravitational eikonal deals with massless interactions)*

Contents

The Simplest Calabi–Yau

Focus on Calabi–Yau 3-folds:

$$h^{r,s} = \dim_{\mathbb{C}} H^{r,s}_{\bar{\partial}}(M, \mathbb{C}) \qquad \Rightarrow \qquad \begin{cases} h^{0,0} & = h^{3,0} = 1 \\ h^{r,0} & = 0 \quad \text{if} \quad r \neq 3 \\ h^{r,s} & = h^{3-r,3-s} \\ h^{1,1},\ h^{2,1} \in \mathbb{N} & \end{cases}$$

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Complete Intersection Calabi–Yau Manifolds

Intersection of hypersurfaces in

$$\mathcal{A} = \mathbb{P}^{n_1} \times \cdots \times \mathbb{P}^{n_m}$$

where

$$\mathbb{P}^n : \qquad \begin{cases} p_a\left(Z^0,\,\ldots,\,Z^n\right) & = P_{I_1 \ldots I_a} Z^{I_1} \ldots Z^{I_a} = 0 \\ p_a\left(\lambda Z^0,\,\ldots,\,\lambda Z^n\right) & = \lambda^a p_a\left(Z^0,\,\ldots,\,Z^n\right) \end{cases}$$

[Green, Hübsch (1987); Hübsch (1992)]

Representation of the Output

CICY can be generalised to m projective spaces and k equations. The problem is thus mapped to:

$$\begin{array}{cccc}
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Machine Learning Approach

What is $\mathscr{R}$?

$$
\mathscr{R}(M) \longrightarrow \mathscr{R}_n(M;\, w) \qquad \text{s.t.} \qquad \lim_{n\to\infty} \left| \mathscr{R}(M) - \mathscr{R}_n(M;\, w) \right| = 0
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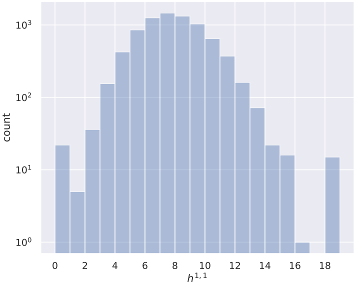
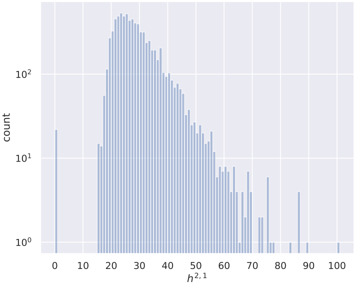
- provide in-depth **data analysis** of the datasets





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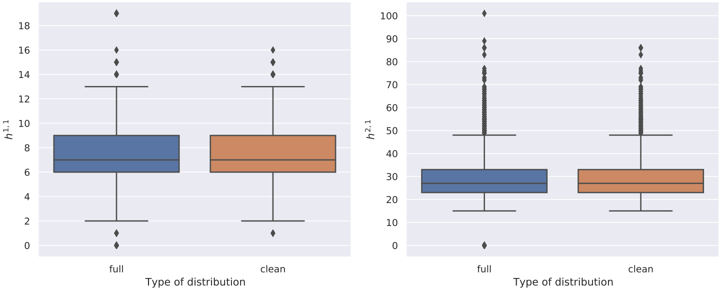

Exploratory Data Analysis

Machine Learning pipeline:

exploratory data analysis $\rightarrow$ feature **selection** $\rightarrow$ Hodge numbers

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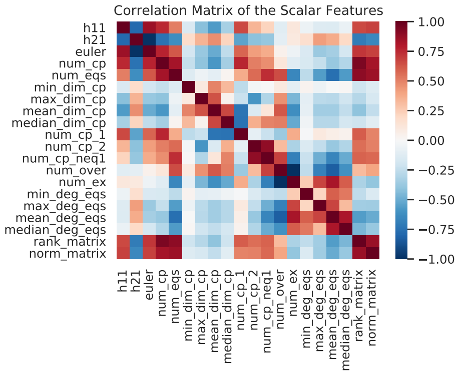
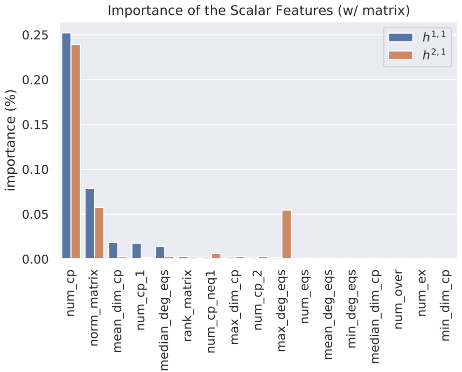
exploratory data analysis → feature **selection** → Hodge numbers





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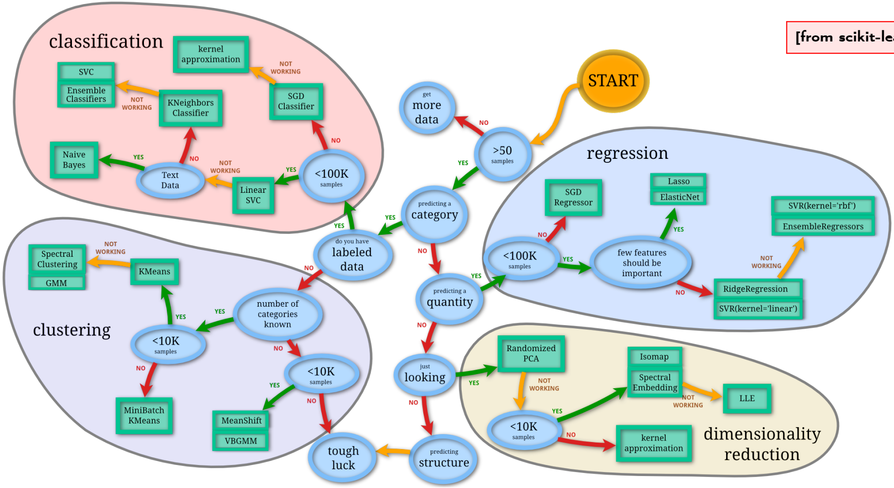

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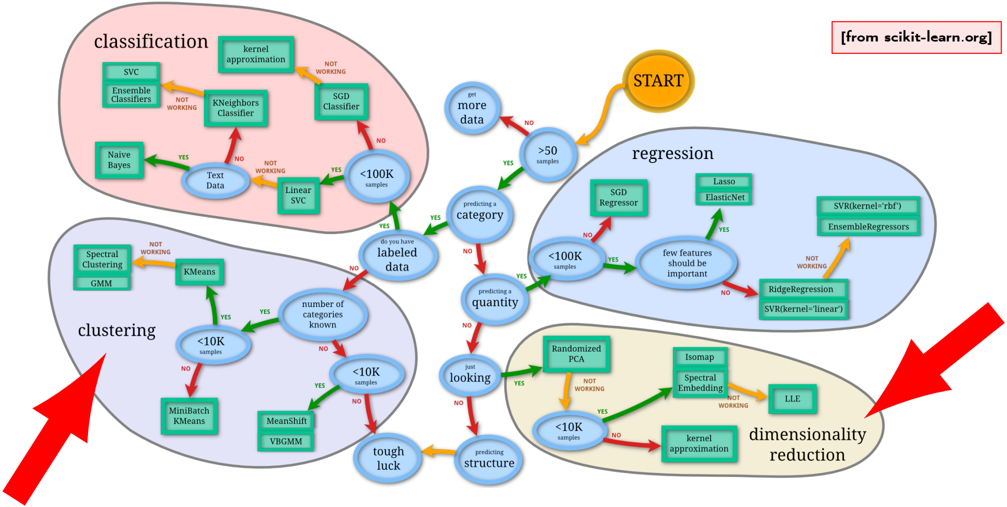

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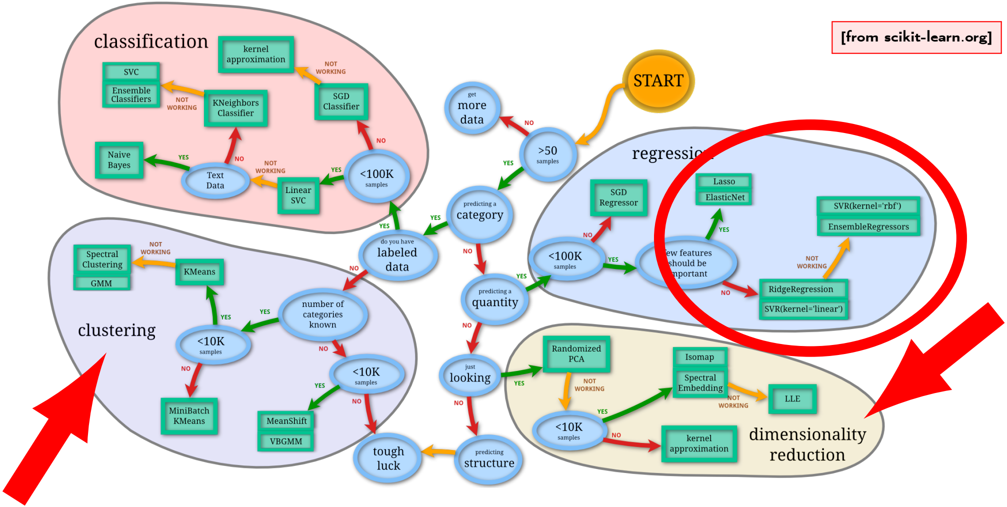

A Word on PCA

What is PCA for a $X \in \mathbb{R}^{n \times p}$?

- project data onto a **lower dimensional** space where variance is maximised
- equivalently compute the **eigenvectors** of XX^T or the **singular values** of X
- isolate **the signal** from the **background**
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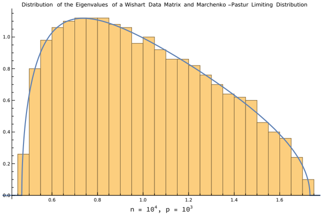
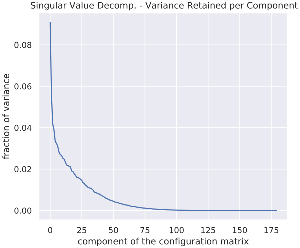
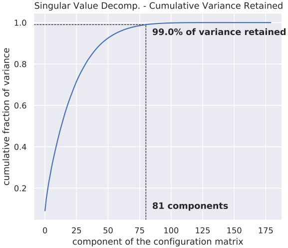
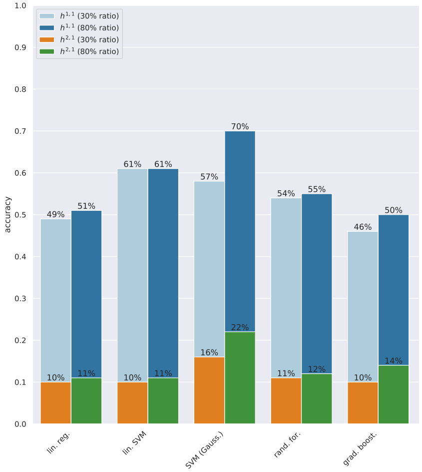






Machine Learning Results

Configuration Matrix Only



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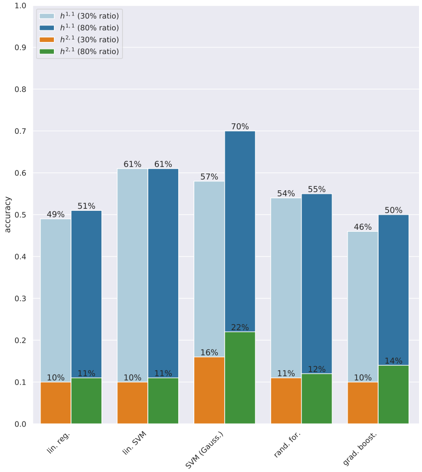
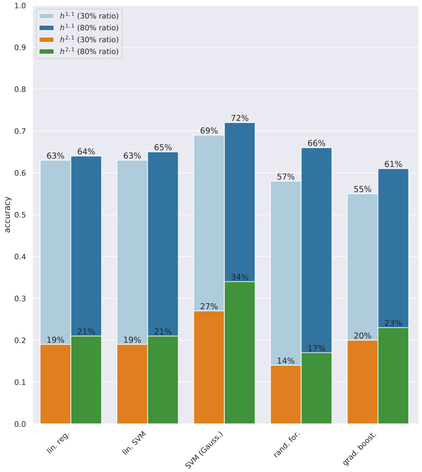


Best Training Set



Artificial Intelligence and Neural Networks

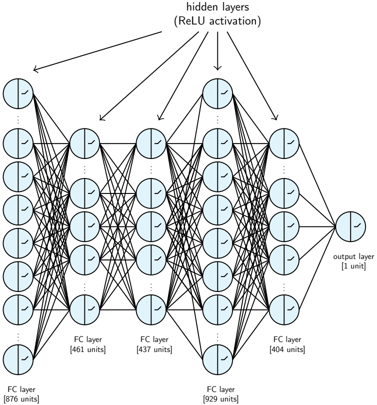
- use **gradient descent** to optimise **weights**
- learn highly **non linear** representations of the input
- can be "large" to have enough parameters
- can be "deep" to to learn **complicated functions**

Layers

fully connected: $\phi\left(a^{(i)\,\{l\}} \cdot W^{\{l\}} + b^{\{l\}}\mathbb{1}_l\right)$
convolutional: $\phi\left(a^{(i)\,\{l\}} * W^{\{l\}} + b^{\{l\}}\mathbb{1}_l\right)$

Non linearity ensured by:

$$\phi(z) = \mathrm{ReLU}(z) = \max\left(0,\, z\right)$$

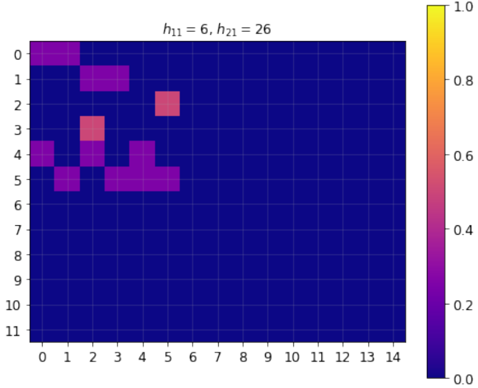


[rendition of the neural network in Bull et al. (2018)]

Convolutional Neural Networks

Why convolutional?

- retain **spacial awareness**



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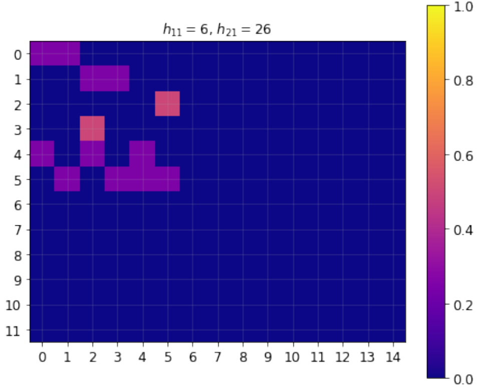

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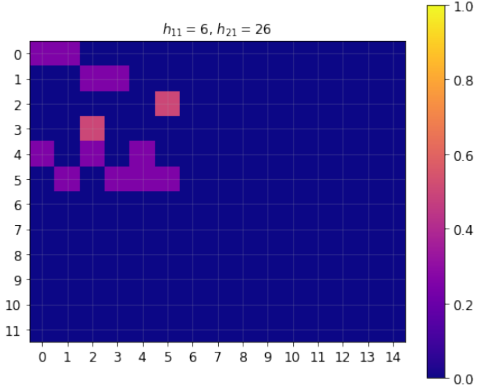

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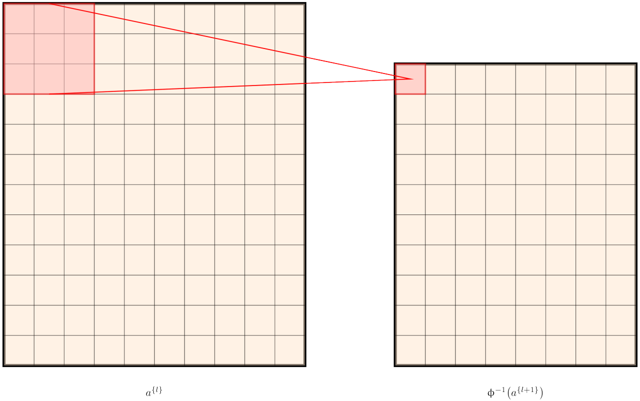

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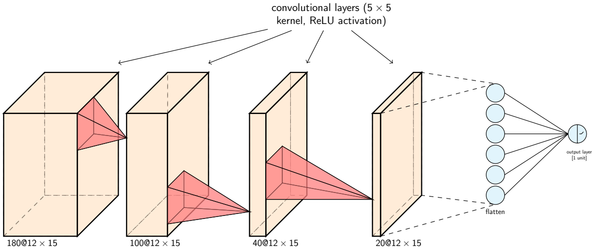
- find patterns as in **computer vision**



Inception Neural Networks

Recent development by Google's deep learning teams led to:

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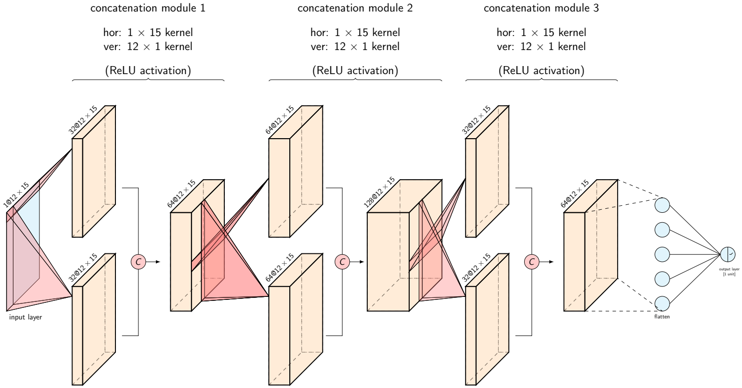

Deep Learning Results and Generalisation Properties

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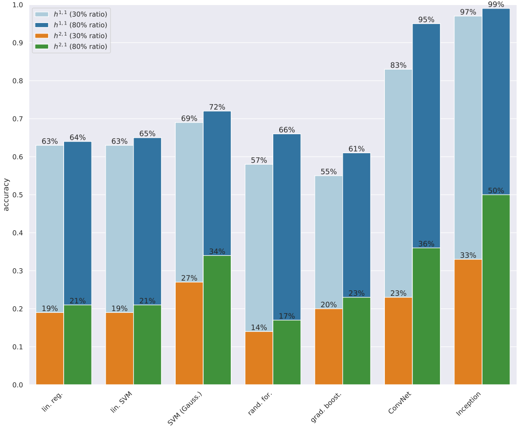

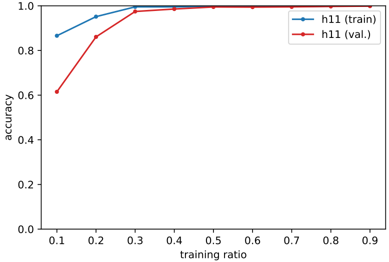

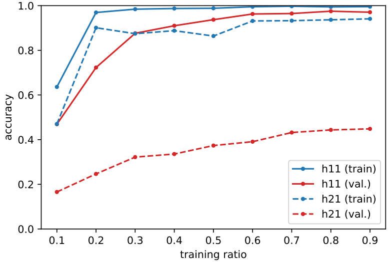

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The End?

THANK YOU