D-branes and Deep Learning

Theoretical and Computational Aspects in String Theory

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15th December 2020

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Polyakov's Action

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Symmetries:

- **Poincaré transf.** $X'^\mu = \Lambda^\mu{}_\nu X^\nu + c^\mu$
- **2D diff.** $\gamma'_{\alpha\beta} = \left(J^{-1}\right)_{\alpha\beta}{}^{\lambda\rho} \gamma_{\lambda\rho}$
- **Weyl transf.** $\gamma'_{\alpha\beta} = e^{2\omega} \gamma_{\alpha\beta}$

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Conformal symmetry:

- **vanishing** stress-energy tensor: $\mathcal{T}_{\alpha\beta} = 0$
- **traceless** stress-energy tensor: $\operatorname{tr} \mathcal{T} = 0$
- **conformal gauge** $\gamma_{\alpha\beta} = e^{\phi} \eta_{\alpha\beta}$

Action Principle and Conformal Symmetry

Superstrings in D dimensions:

$$\mathcal{T}(z) = -\frac{1}{\alpha'}\partial X(z)\cdot\partial X(z) - \frac{1}{2}\psi(z)\cdot\partial\psi(z) \quad\Rightarrow\quad c = \frac{3}{2}D$$

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$(\lambda, 0)$ / $(1-\lambda, 0)$ Ghost System

Introduce anti-commuting $(b,\, c)$ and commuting $(\beta,\, \gamma)$ conformal fields:

$$S_{\mathrm{ghost}}[b,\, c,\, \beta,\, \gamma] = \frac{1}{2\pi}\iint \mathrm{d}z\,\mathrm{d}\overline{z}\,\left(b(z)\,\overline{\partial}c(z) + \beta(z)\,\overline{\partial}\gamma(z)\right)$$

where $\lambda_b = 2$ and $\lambda_c = -1$, and $\lambda_\beta = \frac{3}{2}$ and $\lambda_\gamma = -\frac{1}{2}$.

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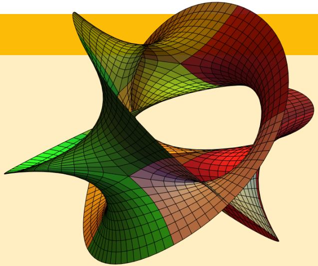
Consequence:

$$c_{\text{full}} = c + c_{\text{ghost}} = 0 \quad\Leftrightarrow\quad D = 10.$$

Extra Dimensions and Compactification

Compactification

$$\mathscr{M}^{1,9} = \mathscr{M}^{1,3} \otimes \mathscr{X}_6$$



- $\mathscr{X}_6$ is a **compact** manifold
- $N = 1$ **supersymmetry** is preserved in 4D
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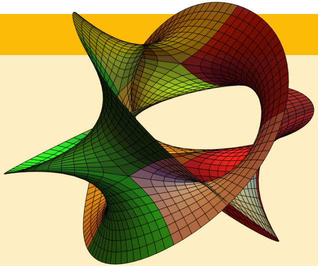

Kähler manifolds (M, g) such that

- $\dim_{\mathbb{C}} M = m$
- $\mathrm{Hol}(g) \subseteq \mathrm{SU}(m)$
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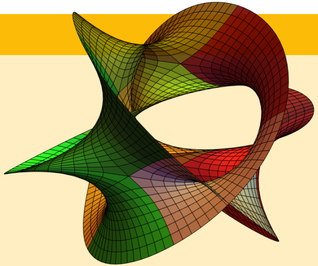

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Characterised by Hodge numbers

$$h^{r,s} = \dim_{\mathbb{C}} H^{r,s}_{\bar\partial}(M, \mathbb{C})$$

counting the no. of harmonic (r, s)-forms.

D-branes and Open Strings

Polyakov's action naturally introduces Neumann b.c.:

$$\partial_\sigma X(\tau, \sigma)\Big|_{\sigma=0}^{\sigma=\ell} = 0$$

satisfied by **open and closed** strings living in D dimensions s.t. $\Box X = 0$.

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T-duality

$$X(z, \bar{z}) = X(z) + \overline{X}(\bar{z}) \quad \overset{T}{\Rightarrow} \quad X(z) - \overline{X}(\bar{z}) = Y(z, \bar{z}) = Y(z) + \overline{Y}(\bar{z})$$

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Resulting effect (repeated $p \leq D-1$ times) leads to Dirichlet b.c.:

$$\partial_\sigma X^i(\tau,\sigma)\Big|_{\sigma=0}^{\sigma=\ell} = 0 \quad \overset{T}{\Rightarrow} \quad \partial_\tau Y^i(\tau,\sigma)\Big|_{\sigma=0}^{\sigma=\ell} = 0 \quad \forall i = 1, 2, \dots, p$$

thus **open strings** can be **constrained** to $D(D-p-1)$-branes.

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$$\mathcal{A}^{\mu} \quad \leftrightarrow \quad \alpha^{\mu}_{-1}\left|0\right\rangle \qquad \longrightarrow \qquad \begin{array}{lcll} \mathcal{A}^{A} & \leftrightarrow & \alpha^{A}_{-1}\left|0\right\rangle, & A = 0,\, 1,\, \dots,\, p \\ \mathcal{A}^{a} & \leftrightarrow & \alpha^{a}_{-1}\left|0\right\rangle, & a = 1,\, 2,\, \dots,\, D-p-1 \end{array}$$

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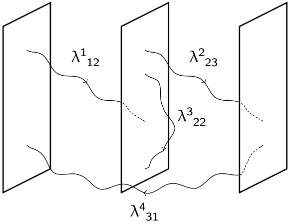

Introducing **Chan–Paton factors** $\lambda^{r}{}_{ij}$, when branes are **coincident**:

$$\bigoplus_{r=1}^{N} \mathrm{U}_r(1) \quad \longrightarrow \quad \mathrm{U}(N)$$

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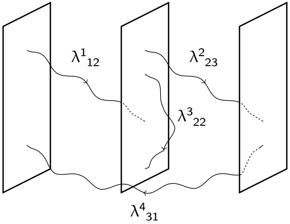

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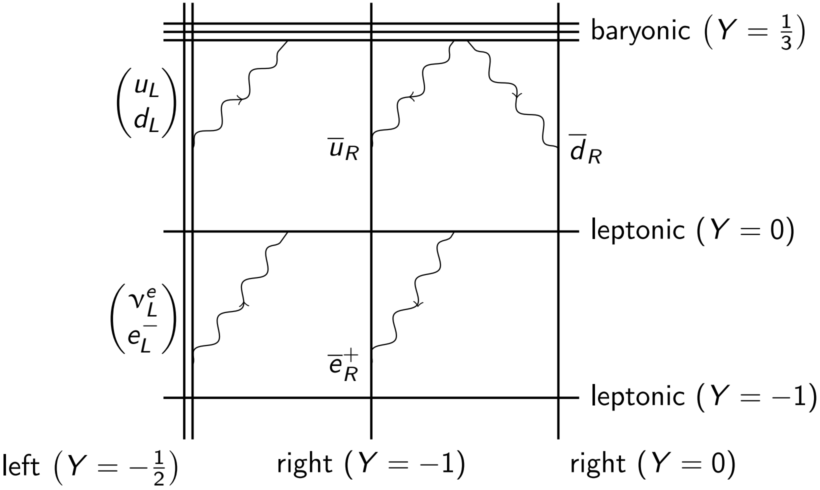
Build gauge bosons, fermions and scalars.

Standard Model-like Scenarios



Intersecting D-branes

Consider N intersecting $D6$-branes filling $\mathscr{M}^{1,3}$ and **embedded in** $\mathbb{R}^6$

Twist Fields Correlators

$$\left\langle \prod_{t=1}^{N_B} \sigma_{\mathrm{M}_{(t)}}\left(x_{(t)}\right) \right\rangle = \mathcal{N}\left(\left\{ x_{(t)},\, \mathrm{M}_{(t)} \right\}_{1 \leq t \leq N_B} \right) e^{-S_{E\,(\mathbf{cl})}\left(\left\{ x_{(t)},\, \mathrm{M}_{(t)} \right\}_{\mathbf{1} \leq t \leq N_B} \right)}$$

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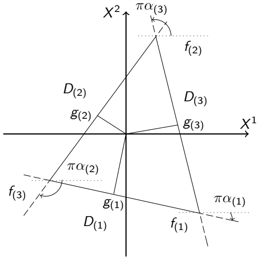

D-branes in **factorised** internal space:

- **embedded as lines** in $\mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2$
- **relative rotations** are $\mathrm{SO}(2) \simeq \mathrm{U}(1)$ elements
- $S_{E\,(\mathrm{cl})}\left(\left\{x_{(t)},\, \mathrm{M}_{(t)}\right\}_{1 \le t \le N_B}\right) \sim \mathrm{Area}\left(\left\{f_{(t)},\, \mathrm{R}_{(t)}\right\}_{1 \le t \le N_B}\right)$

$\mathrm{SO}(4)$ Rotations

Consider $\boxed{\mathbb{R}^4 \times \mathbb{R}^2}$ (focus on $\mathbb{R}^4$):

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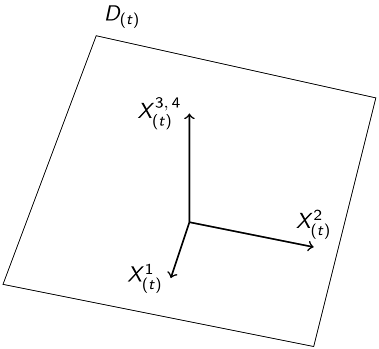

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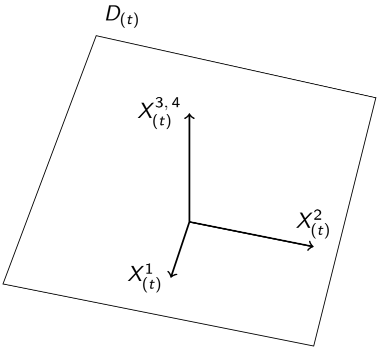

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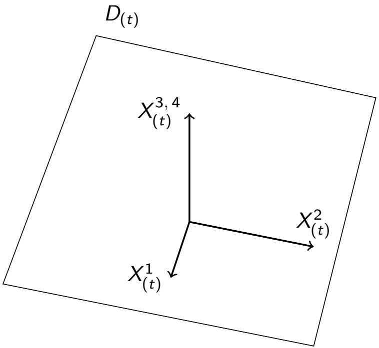

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that is

$$\left[R_{(t)}\right] = \left\{R_{(t)} \sim \mathcal{O}_{(t)} R_{(t)}\right\}$$

Boundary Conditions

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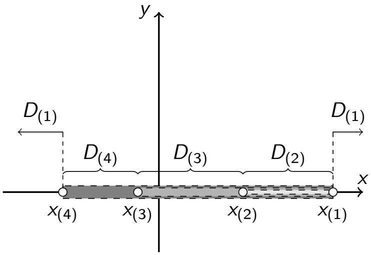
- consider $u = x + iy = e^{\tau_e + i\sigma}$ and $\overline{u} = u^*$
- let $x_{(t)} < x_{(t-1)}$ be the **worldsheet intersection points** on **real axis**
- $X^{1,2}_{(t)}$ are **Neumann**, $X^{3,4}_{(t)}$ are **Dirichlet**



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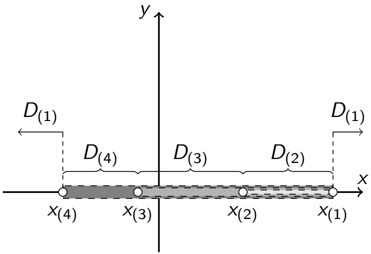

Branch Cuts and Discontinuities for $x \in D_{(t)}$

$$\begin{cases} \partial_u X(x + i0^+) & = U_{(t)} \cdot \partial_{\overline{u}} \overline{X}(x - i0^+) = \left[R^{-1}_{(t)} \cdot (\sigma_3 \otimes \mathbb{1}_2) \cdot R_{(t)} \right] \cdot \partial_{\overline{u}} \overline{X}(x - i0^+) \\ X(x_{(t)}, x_{(t)}) & = f_{(t)} \end{cases}$$

Doubling Trick and Spinor Representation

Doubling Trick

$$\partial_z \mathcal{X}(z) = \begin{cases} \partial_u X(u) & \text{if } z \in \mathscr{H}_{>}^{(\bar{t})} \\ U_{(\bar{t})}\, \partial_{\bar{u}} \overline{X}(\bar{u}) & \text{if } z \in \mathscr{H}_{<}^{(\bar{t})} \end{cases} \quad \Rightarrow \quad \begin{aligned} \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i}\delta_+) &= \mathcal{U}_{(t,\,t+1)}\, \partial_z \mathcal{X}(x_{(t)} + \delta_+), \\ \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i}\delta_-) &= \widetilde{\mathcal{U}}_{(t,\,t+1)}\, \partial_z \mathcal{X}(x_{(t)} + \delta_-), \end{aligned}$$

where $\mathscr{H}_{\gtrless}^{(t)} = \left\{ z \in \mathbb{C} \mid \operatorname{Im} z \gtrless 0 \text{ or } z \in D_{(t)} \right\}$ and $\delta_\pm = \eta \pm i0^+$.

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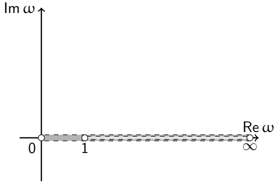
Use Pauli matrices $\tau = (i\,\mathbb{1}_2, \vec{\sigma})$:

$$\partial_z \mathcal{X}_{(s)}(z) = \partial_z \mathcal{X}^I(z)\, \tau_I \quad \Rightarrow \quad \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i}\,\delta_\pm) = \overset{(\sim)}{\mathcal{L}}_{(t,\,t+1)}\, \partial_z \mathcal{X}(x_{(t)} + \delta_\pm)\, \overset{(\sim)}{\mathcal{R}}_{(t,\,t+1)}$$

where

$$\overset{(\sim)}{\mathcal{L}}_{(t,\,t+1)} \in \mathrm{SU}(2)_L \quad \text{and} \quad \overset{(\sim)}{\mathcal{R}}_{(t,\,t+1)} \in \mathrm{SU}(2)_R$$

Hypergeometric Basis



Sum over all contributions:

$$\partial_z \mathcal{X}(z) = \sum_{l,\, r} c_{lr}\, (-\omega_z)^{A_{lr}}\, (1-\omega_z)^{B_{lr}}\, B^{(L)}_{0,\, l}(\omega_z) \left(B^{(R)}_{0,\, r}(\omega_z) \right)^T$$

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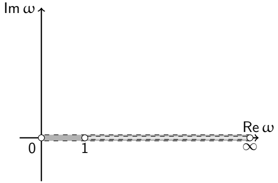

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Basis of Solutions

$$B_{0,\,n}(\omega_z) = \begin{pmatrix} 1 & 0 \\ 0 & K_n \end{pmatrix} \begin{pmatrix} \frac{1}{\Gamma(c_n)}\,{}_2F_1\left(a_n,\, b_n;\, c_n;\, \omega_z\right) \\ \frac{(-\omega_z)^{1-c_n}}{\Gamma(2-c_n)}\,{}_2F_1\left(a_n+1-c_n,\, b_n+1-c_n;\, 2-c_n;\, \omega_z\right) \end{pmatrix}$$

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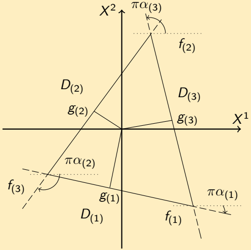
Physical Interpretation



$$S_{\mathbb{R}^4}\Big|_{\text{on-shell}} = \frac{1}{2\pi\alpha'}\sum_{t=1}^{3}\left(\frac{1}{2}\left|g_{(t)}^{\perp}\right|\left|f_{(t-1)} - f_{(t)}\right|\right) = \text{Area}\left(\left\{f_{(t)}\right\}\right)$$

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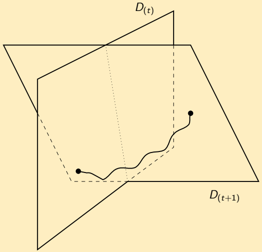
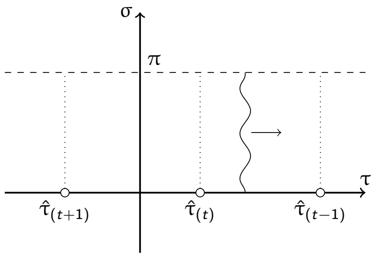
Physical Interpretation



Fermions on the Strip



Action of Boundary Changing Operators

$$\begin{cases} \psi^i_-(\tau,0) & = \left(R_{(t)}\right)^I{}_J\,\psi^J_+(\tau,0) \quad \text{for } \tau \in \left(\hat\tau_{(t)},\, \hat\tau_{(t-1)}\right) \\ \psi^I_-(\tau,\pi) & = -\psi^I_+(\tau,\pi) \quad \text{for } \tau \in \mathbb{R} \end{cases}$$

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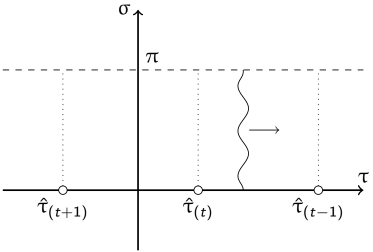

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Stress-energy Tensor

$$\mathcal{T}_{\pm\pm}(\xi_\pm) = -i\,\frac{T}{4}\,\psi^*_{\pm,\,I}(\xi_\pm)\,\overset{\leftrightarrow}{\partial}\psi^I_\pm(\xi_\pm) \quad \Rightarrow \quad \begin{cases} \dot{\mathrm{H}}(\tau) &= 0 \Leftrightarrow \tau \in \left(\tau_{(t)},\, \tau_{(t-1)}\right) \\ \dot{\mathrm{P}}(\tau) &\neq 0 \end{cases}$$

Conserved Product and Operators

Expand on a basis of solutions

$$\psi_\pm(\xi_\pm) = \sum_{n=-\infty}^{+\infty} b_n\, \psi_n(\xi_\pm) \qquad \Rightarrow \qquad \Psi(z) = \begin{cases} \psi_{E,+}(u) & \text{if } z \in \mathscr{H}_>^{(\bar{t})} \\ \psi_{E,-}(u) & \text{if } z \in \mathscr{H}_<^{(\bar{t})} \end{cases}$$

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Conserved Product and Dual Basis

$$\langle\!\langle {}^*\psi_n, \psi_m \rangle = 2\pi\mathcal{N} \oint \frac{\mathrm{d}z}{2\pi i} \, {}^*\Psi_n \, {}^*\Psi_m = \delta_{n,m} \qquad \Rightarrow \qquad \Big\langle\!\Big\langle {}^*\Psi_n{}^{(*)}, \Psi^{(*)} \Big\rangle = b_n^{(\dagger)}$$

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Derive the algebra of operators:

$$\left[b_n, b_m^\dagger\right]_+ = \frac{2\mathcal{N}}{T} \, \langle\!\langle {}^*\Psi_n{}^*, \Psi_m^* \rangle$$

Twisted Complex Fermions

Consider the case $R_{(t)} = e^{i\pi\alpha_{(t)}} \in \mathrm{U}(1)$:

$$\Psi(x_{(t)} + e^{2\pi i}\delta) = e^{i\pi\epsilon_{(t)}}\Psi(x_{(t)} + \delta)$$

where

$$\epsilon_{(t)} = \alpha_{(t+1)} - \alpha_{(t)} + \theta\left(\alpha_{(t)} - \alpha_{(t+1)} - 1\right) - \theta\left(\alpha_{(t+1)} - \alpha_{(t)} - 1\right)$$

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Basis of Solutions

$$\Psi_n\big(z;\,\{x_{(t)}\}\big) = \mathcal{N}_\Psi\, z^{-n} \prod_{t=1}^{N} \left(1 - \frac{z}{x_{(t)}}\right)^{n_{(t)} + \frac{\epsilon_{(t)}}{2}}$$

$${}^{*}\Psi_n\big(z;\,\{x_{(t)}\}\big) = \frac{1}{2\pi\mathcal{N}\mathcal{N}_\Psi}\, z^{n-1} \prod_{t=1}^{N} \left(1 - \frac{z}{x_{(t)}}\right)^{-\widetilde{n}_{(t)} + \frac{\epsilon_{(t)}}{2}}$$

Vacua

Define the **vacuum** with respect to b_n:

$$b_n \left| \left\{ x_{(t)} \right\} \right\rangle = 0 \quad \text{for} \quad n \geq 1$$

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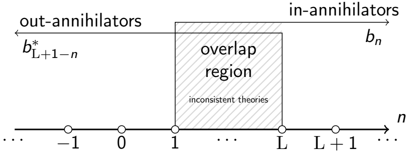
Theories are subject to consistency conditions:

$$\mathrm{L} = n_{(t)} + \widetilde{n}_{(t)}$$



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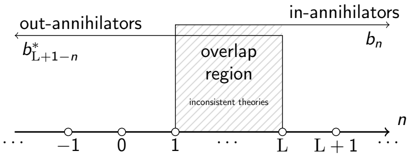

Stress-energy Tensor and CFT Approach

Compute the OPEs leading to the stress-energy tensor:

$$\mathcal{T}(z) = \frac{\pi T}{2} \mathcal{N}_\Psi^2 \sum_{n,\, m=-\infty}^{+\infty} :b_n\, b_m^*:\; z^{-n-m} \left[\frac{m-n}{2} + 2 \sum_{t=1}^{N} \frac{n_{(t)} + \frac{\epsilon_{(t)}}{2}}{z - x_{(t)}} \right] + \frac{1}{2} \left(\sum_{t=1}^{N} \frac{n_{(t)} + \frac{\epsilon_{(t)}}{2}}{z - x_{(t)}} \right)^2$$

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Invariant Vacuum and Spin Fields

$$\left| \left\{ x_{(t)} \right\} \right\rangle = \mathcal{N}\left(\left\{ x_{(t)} \right\} \right) \mathrm{R} \left[\prod_{t=1}^{M} S_{(t)}(x_{(t)}) \right] |0\rangle_{\mathrm{SL}_2(\mathbb{R})}$$

Spin Fields Amplitudes

Equivalence with Bosonization

$$\partial_{x_{(t)}} \left\langle \left\{ x_{(t)} \right\} \middle| \left\{ x_{(t)} \right\} \right\rangle = \oint\limits_{x_{(t)}} \frac{\mathrm{d}z}{2\pi i} \frac{\left\langle \left\{ x_{(t)} \right\} \middle| \mathcal{T}(z) \middle| \left\{ x_{(t)} \right\} \right\rangle}{\left\langle \left\{ x_{(t)} \right\} \middle| \left\{ x_{(t)} \right\} \right\rangle}$$

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- alternative framework for amplitudes (extension to (non) Abelian twist/spin fields?)

Contents

A Few Words on a Theory of Everything

string theory = theory of everything = nuclear forces + gravity

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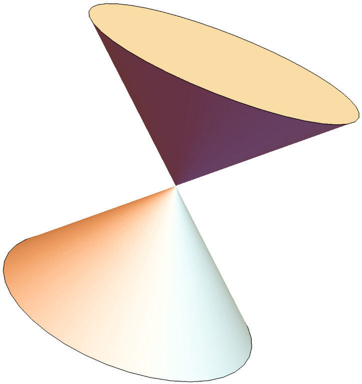

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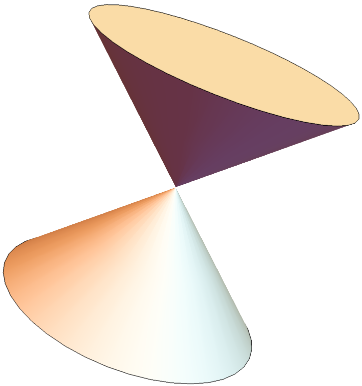

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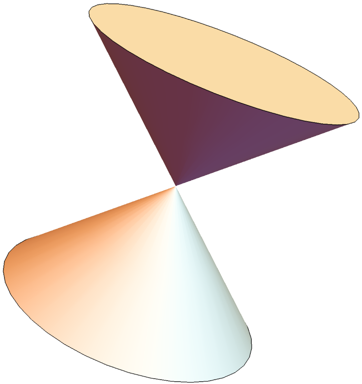

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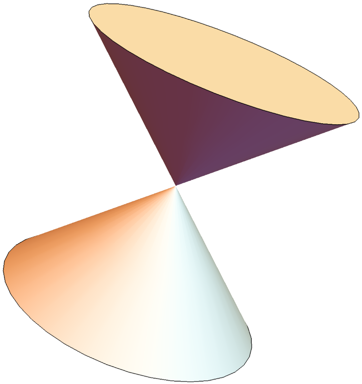

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$$\Downarrow$$

time-dependent orbifold models

Orbifolds

Mathematics

- manifold M
- (Lie) group G
- *stabilizer* $G_p = \{g \in G \mid gp = p \in M\}$
- *orbit* $Gp = \{gp \in M \mid g \in G\}$
- charts $\phi = \pi \circ \mathscr{P}$ where:
  - $\mathscr{P} \colon U \subset \mathbb{R}^n \to U/G$
  - $\pi \colon U/G \to M$

$\Rightarrow$

Physics

- global orbit space M/G
- G group of isometries
- fixed points
- additional d.o.f. (*twisted states*)
- singular limits of CY manifolds

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Cosmological Singularities

Use **time-dependent orbifolds** to model **space-like singularities**:

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Cosmological Singularities

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Divergences

Even in simple models (e.g. NBO, more on this later) the 4 tachyons amplitude is divergent **at tree level**:

$$A_4 \sim \int_{q\sim\infty} \frac{\mathrm{d}q}{|q|} \mathscr{A}(q)$$

where

$$\mathscr{A}_{\text{closed}}(q) \sim q^{4-\alpha' \|\vec{p}_\perp\|^2} \qquad \text{and} \qquad \mathscr{A}_{\text{open}}(q) \sim q^{1-\alpha' \|\vec{p}_\perp\|^2} \operatorname{tr}([T_1,\, T_2]_+ [T_3,\, T_4]_+)$$

Null Boost Orbifold

Start from $\left(x^+,\, x^-,\, x^2,\, \vec{x}\right) \in \mathscr{M}^{1,\, D-1}$:

$$\begin{cases} u &= x^- \\ z &= \frac{x^2}{\Delta\, x^-} \\ v &= x^+ - \frac{1}{2} \frac{\left(x^2\right)^2}{x^-} \end{cases} \quad \Rightarrow \quad \mathrm{d}s^2 = -2\, \mathrm{d}u\, \mathrm{d}v + \left(\Delta\, u\right)^2\, \mathrm{d}z^2 + \delta_{ij}\, \mathrm{d}x^i\, \mathrm{d}x^j$$

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$$\kappa = -i(2\pi\Delta)J_{+2} = 2\pi\partial_z \Rightarrow z \sim z + 2\pi n$$

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Killing Vector and Null Boost Oribfold

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Consider scalar QED:

$$\phi_{\{k_+, l, \vec{k}, r\}}(u, v, z, \vec{x}) = e^{i\left(k_+ v + lz + \vec{k}\cdot\vec{x}\right)}\,\widetilde{\phi}_{\{k_+, l, \vec{k}, r\}}(u) = \frac{e^{i\left(k_+ v + lz + \vec{k}\cdot\vec{x}\right)}}{\sqrt{(2\pi)^D\,|2\Delta k_+ u|}}\, e^{-i\frac{l^2}{2\Delta^2 k_+}\frac{1}{u} + i\frac{\|\vec{k}\|^2 + r}{2k_+}u}$$

Scalar QED Interactions

Scalar–photon interactions:

$$S_{\mathrm{sQED}}^{(\mathrm{int})} = \int\limits_{\Omega} \mathrm{d}^D x \, \sqrt{-g} \left(-i \, e \, g^{\alpha\beta} a_\alpha (\phi^* \, \partial_\beta \phi - \partial_\beta \phi^* \, \phi) + e^2 \, g^{\alpha\beta} a_\alpha a_\beta |\phi|^2 - \frac{g_4}{4} |\phi|^4 \right)$$

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Terms involved:

$$\mathcal{I}^{[\nu]}_{\{N\}} = \int\limits_{-\infty}^{+\infty} \mathrm{d}u\, |\Delta\, u| u^\nu \prod_{i=1}^{N} \widetilde{\phi}_{\{k_{+\,(i)},\, l_{(i)},\, \vec{k}_{(i)},\, r_{(i)}\}}(u)$$

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most terms ***do not converge*** *and cannot be recovered even with a* ***distributional interpretions*** *due to the term $\propto u^{-1}$ in the exponentatial*

String and Field Theory

So far:

- field theory presents **divergences**

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Massive String States

$$V_M(x;\, k,\, S,\, \xi) =: \left(\frac{i}{\sqrt{2\alpha'}}\, \xi \cdot \partial_x^2 X(x,\, x) + \left(\frac{i}{\sqrt{2\alpha'}} \right)^2 S_{\alpha\beta} \partial_x X^{\alpha}(x,\, x) \partial_x X^{\beta}(x,\, x) \right) e^{ik \cdot X(x,\, x)} :$$

String and Field Theory

So far:

- field theory presents **divergences**
- issues are **still present** in sQED (eikonal?)
- divergences are **not (only) gravitational**

Massive String States

$$V_M(x;\, k,\, S,\, \xi) =: \left(\frac{i}{\sqrt{2\alpha'}}\, \xi \cdot \partial_x^2 X(x,\, x) + \left(\frac{i}{\sqrt{2\alpha'}} \right)^2 S_{\alpha\beta} \partial_x X^{\alpha}(x,\, x) \partial_x X^{\beta}(x,\, x) \right) e^{ik\cdot X(x,\, x)} :$$

*string theory cannot do **better than field theory** (EFT) if the latter **does not exist** (even a Wilson line around z does not prevent such behaviour)*

Resolution and Motivation

Introduce the generalised NBO:

$$\begin{cases} u &= x^- \\ z &= \frac{1}{2x^-}\left(\frac{x^2}{\Delta_2} + \frac{x^3}{\Delta_3}\right) \\ w &= \frac{1}{2x^-}\left(\frac{x^2}{\Delta_2} - \frac{x^3}{\Delta_3}\right) \\ v &= x^+ - \frac{1}{2x^-}\left(\left(x^2\right)^2 + \left(x^3\right)^2\right) \end{cases} \quad \Rightarrow \quad \kappa = -2\pi i(\Delta_2 J_{+2} + \Delta_3 J_{+3}) = 2\pi\partial_z$$

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Distributional Interpretation

$$\widetilde{\phi}_{\{k_+,\, p,\, l,\, \vec{k},\, r\}}(u) = \frac{1}{2\sqrt{(2\pi)^D |\Delta_2 \Delta_3 k_+|}} \frac{1}{|u|} e^{-i\left(\frac{1}{8k_+ u}\left[\frac{(l+p)^2}{\Delta_2^2} + \frac{(l-p)^2}{\Delta_3^2}\right] - \frac{\|\vec{k}\|^2 + r}{2k_+} u\right)}$$

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spacetime singularities are ***hidden into contact terms*** *and interactions with* ***massive states***
(the gravitational eikonal deals with massless interactions)

Contents

The Simplest Calabi–Yau

Focus on Calabi–Yau 3-folds:

$$h^{r,\,s} = \dim_{\mathbb{C}} H_{\bar{\partial}}^{r,\,s}(M,\,\mathbb{C}) \qquad \Rightarrow \qquad \begin{cases} h^{0,\,0} & = h^{3,\,0} = 1 \\ h^{r,\,0} & = 0 \quad \text{if} \quad r \neq 3 \\ h^{r,\,s} & = h^{3-r,\,3-s} \\ h^{1,\,1},\ h^{2,\,1} \in \mathbb{N} \end{cases}$$

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Complete Intersection Calabi–Yau Manifolds

Intersection of hypersurfaces in

$$\mathcal{A} = \mathbb{P}^{n_1} \times \cdots \times \mathbb{P}^{n_m}$$

where

$$\mathbb{P}^n: \qquad \begin{cases} p_a\left(Z^0, \ldots, Z^n\right) & = P_{I_1 \ldots I_a} Z^{I_1} \ldots Z^{I_a} = 0 \\ p_a\left(\lambda Z^0, \ldots, \lambda Z^n\right) & = \lambda^a p_a\left(Z^0, \ldots, Z^n\right) \end{cases}$$

Representation of the Output

CICY can be generalised to m projective spaces and k equations. The problem is thus mapped to:

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Machine Learning Approach

What is $\mathscr{R}$?

$$\mathscr{R}(M) \longrightarrow \mathscr{R}_n(M;\, w) \qquad \text{s.t.} \qquad \lim_{n\to\infty} f(M;\, w) = \lim_{n\to\infty} \left|\mathscr{R}(M) - \mathscr{R}_n(M;\, w)\right| = 0$$

Machine Learning

- exchange **analytical solution** with **optimisation problem**

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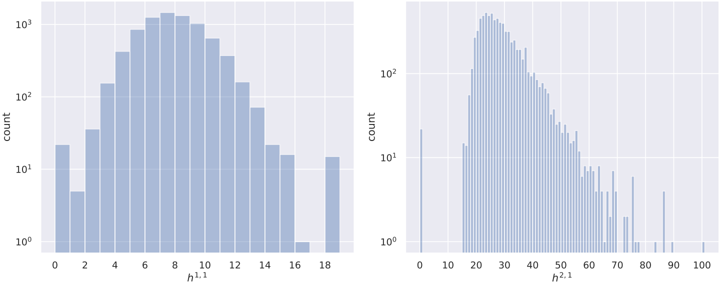

Exploratory Data Analysis

Machine Learning $\boxed{\text{pipeline:}}$

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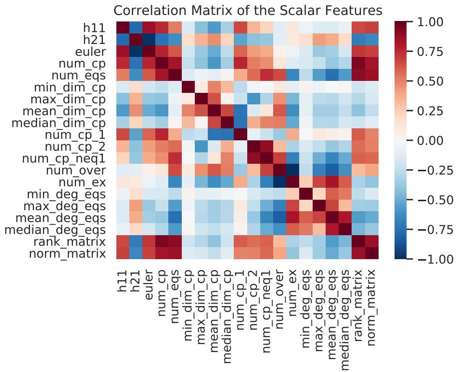

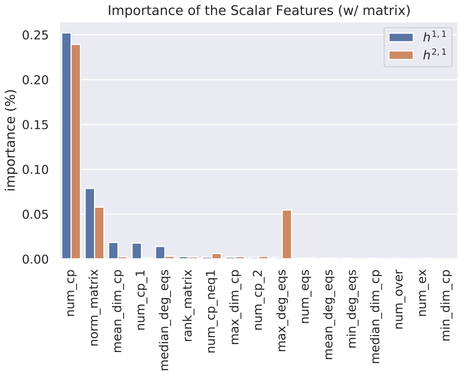

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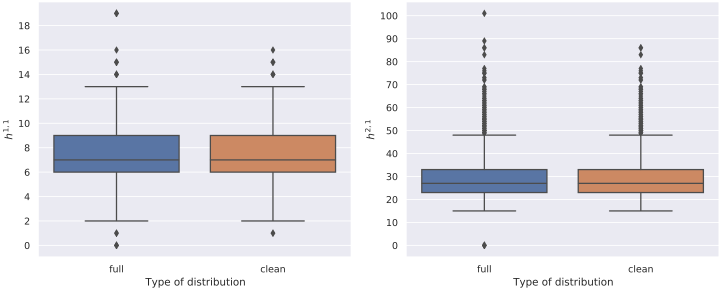
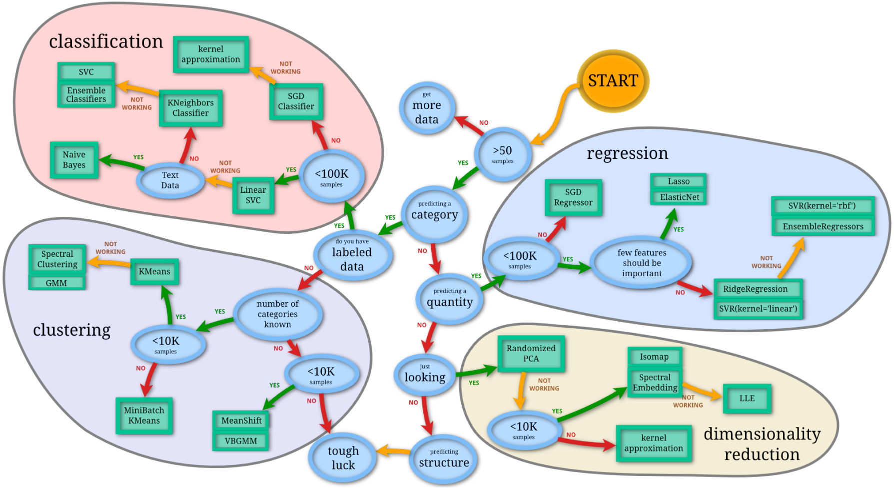


Machine Learning



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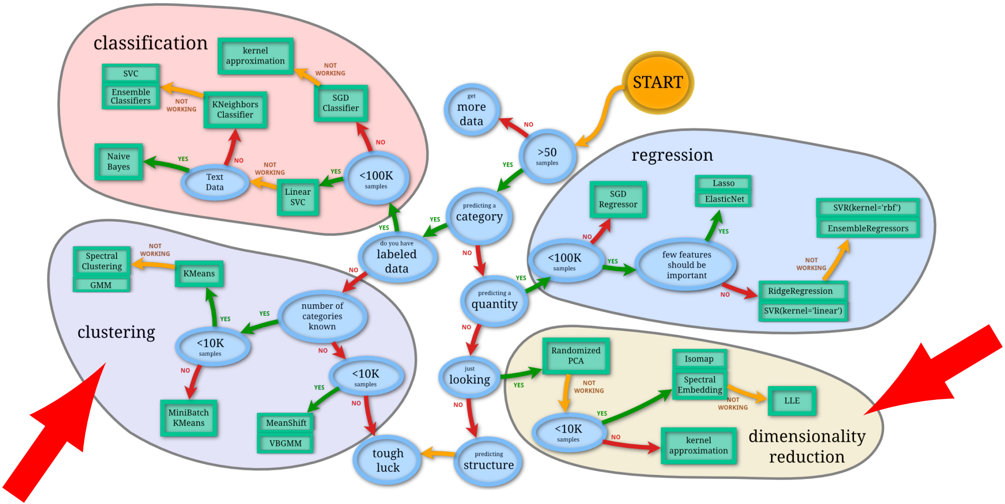

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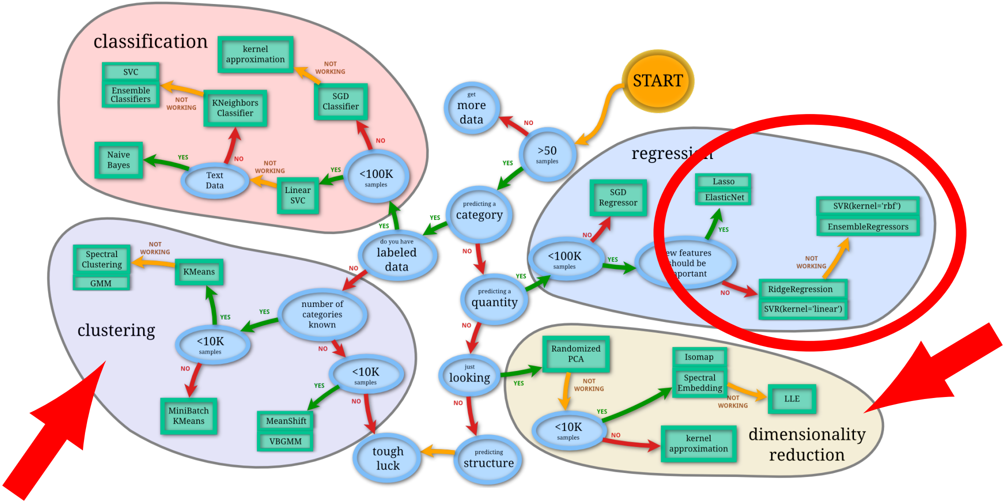
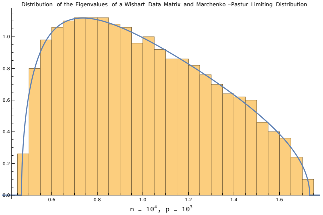
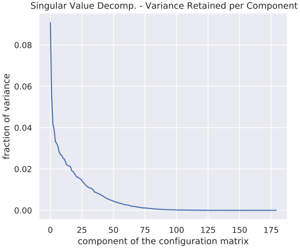


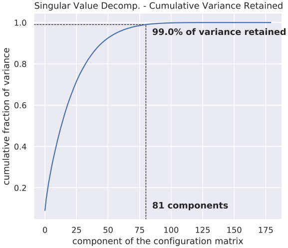
A Word on PCA

What is PCA for a $X \in \mathbb{R}^{n \times p}$?

- find new coordinates to **"put the variance in order"**
- equivalently compute the **eigenvectors** of XX^T or the **singular values** of X
- isolate **the signal** from the **background**
- ease the machine learning job of finding a better representation of the input

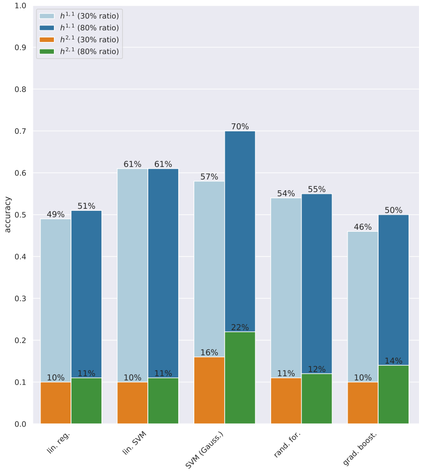






Machine Learning Results

Configuration Matrix Only



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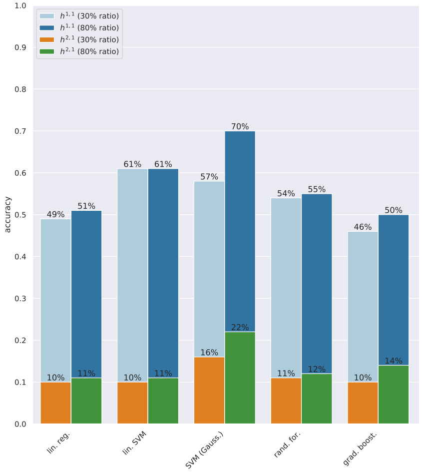
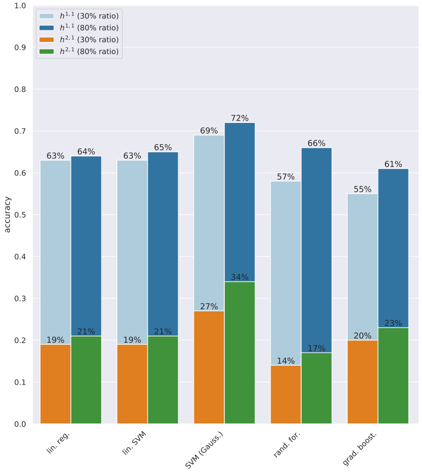


Best Training Set



Artificial Intelligence and Neural Networks

- use **gradient descent** to optimise **weights**
- learn highly **non linear** representations of the input
- can be "large" to have enough parameters
- can be "deep" to to learn **complicated functions**

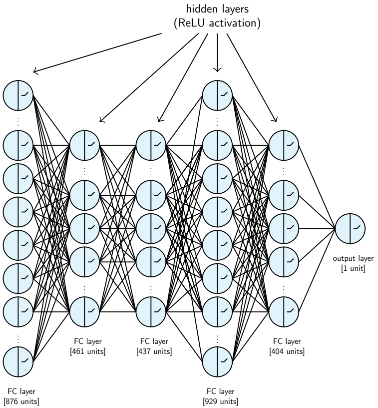
Neural Networks

fully connected: $a^{(i)\,\{l+1\}} = \phi\left(a^{(i)\,\{l\}} \cdot W^{\{l\}} + b^{\{l\}}\mathbb{1}\right)$

convolutional: $a^{(i)\,\{l+1\}} = \phi\left(a^{(i)\,\{l\}} * W^{\{l\}} + b^{\{l\}}\mathbb{1}\right)$

Non linearity ensured by:

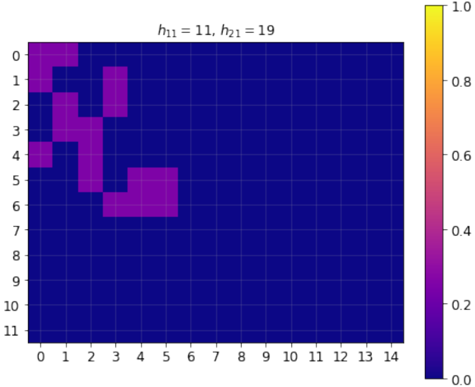
$$\phi(z) = \mathrm{ReLU}(z) = \max\left(0,\, z\right)$$



Convolutional Neural Networks

Why convolutional?

- retain **spacial awareness**



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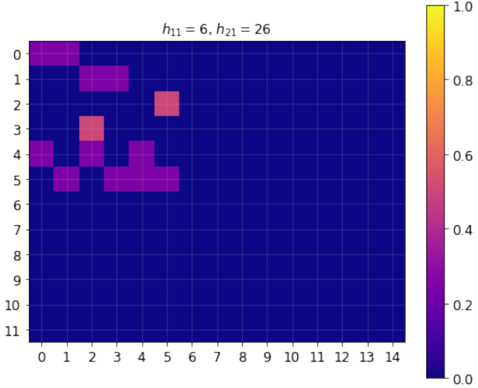

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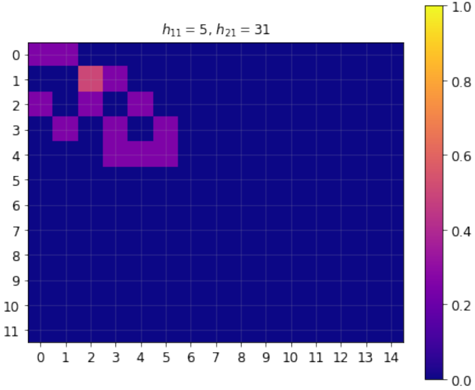

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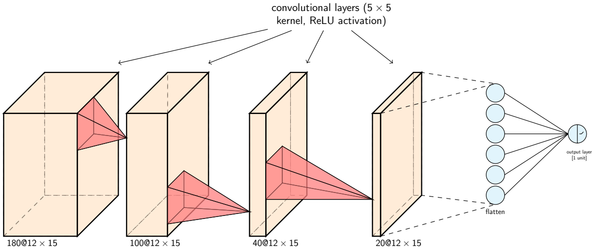

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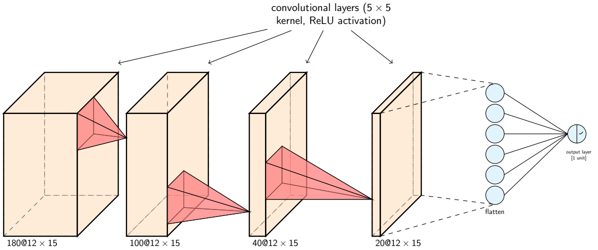
- find patterns as in **computer vision**



Inception Neural Networks

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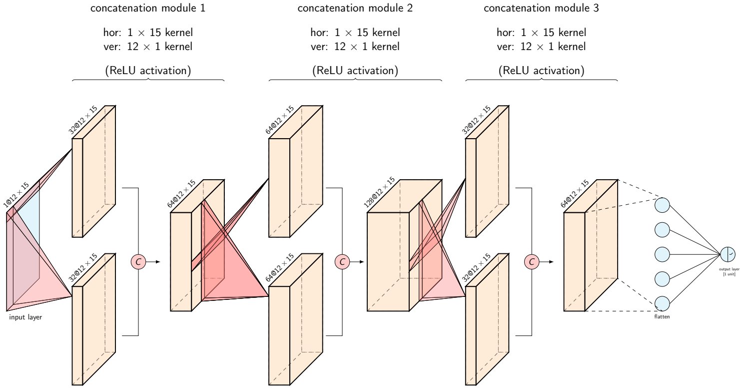
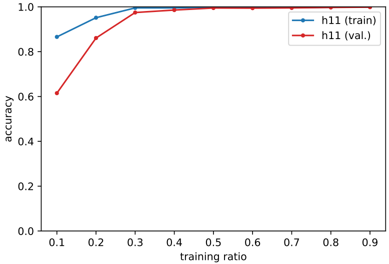
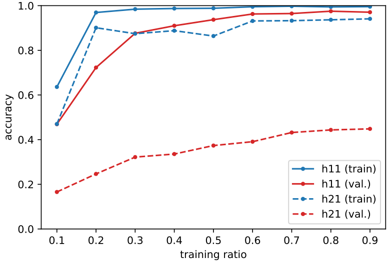


Deep Learning Results and Generalisation Properties





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The End?

THANK YOU!