D-branes and Deep Learning

Theoretical and Computational Aspects in String Theory

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Contents

Contents

Action Principle and Conformal Symmetry

Polyakov's Action

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Action Principle and Conformal Symmetry

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Symmetries:

Poincaré transf.:	$X'^\mu = \Lambda^\mu{}_\nu X^\nu + c^\mu$
2D diff.:	$\gamma'_{\alpha\beta} = \left(J^{-1}\right)_{\alpha\beta}{}^{\lambda\rho} \gamma_{\lambda\rho}$
Weyl transf.:	$\gamma'_{\alpha\beta} = e^{2\omega} \gamma_{\alpha\beta}$

Conformal symmetry:

vanishing stress-energy tensor:	$\mathcal{T}_{\alpha\beta} = 0$
traceless stress-energy tensor:	$\operatorname{tr} \mathcal{T} = 0$
conformal gauge:	$\gamma_{\alpha\beta} = e^{\phi} \eta_{\alpha\beta}$

Action Principle and Conformal Symmetry

Superstrings in D dimensions $\longrightarrow$ *Virasoro algebra* (central extension of de Witt's algebra):

$$\mathcal{T}(z) = -\frac{1}{\alpha'}\partial X(z) \cdot \partial X(z) - \frac{1}{2}\psi(z) \cdot \partial\psi(z) \quad \Rightarrow \quad c = \frac{3}{2}D$$

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$(\lambda, 0)$ / $(1-\lambda, 0)$ Ghost System

Introduce anti-commuting $(b,\ c)$ and commuting $(\beta,\ \gamma)$ conformal fields:

$$S_{\text{ghost}}[b,\ c,\ \beta,\ \gamma] = \frac{1}{2\pi}\iint \mathrm{d}z\,\mathrm{d}\bar{z}\,\left(b(z)\,\bar{\partial}c(z) + \beta(z)\,\bar{\partial}\gamma(z)\right)$$

where $\lambda_b = 2$ and $\lambda_c = -1$, and $\lambda_\beta = \frac{3}{2}$ and $\lambda_\gamma = -\frac{1}{2}$.

[Friedan, Martinec, Shenker (1986)]

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Consequence:

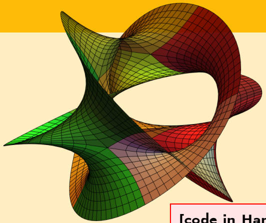
$$c_{\text{full}} = c + c_{\text{ghost}} = 0 \quad \Leftrightarrow \quad D = 10.$$

Extra Dimensions and Compactification

Compactification

$$\mathscr{M}^{1,9} = \mathscr{M}^{1,3} \otimes \mathscr{X}_6$$

- $\mathscr{X}_6$ is a **compact** manifold
- $N = 1$ **supersymmetry** preserved in 4D
- contains algebra of $\mathrm{SU}(3) \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1)$



[code in Hanson (1994)]

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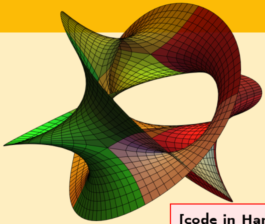

[code in Hanson (1994)]

Calabi–Yau manifolds (M, g) such that:

- $\dim_{\mathbb{C}} M = m$
- $\mathrm{Hol}(g) \subseteq \mathrm{SU}(m)$
- $\mathrm{Ric}(g) \equiv 0$ (equiv. $c_1(M) \equiv 0$)

[Calabi (1957), Yau (1977), Candelas *et al.* (1985)]

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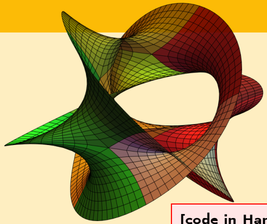

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[Calabi (1957), Yau (1977), Candelas *et al.* (1985)]

Characterised by **Hodge numbers**

$$h^{r,s} = \dim_{\mathbb{C}} H^{r,s}_{\overline{\partial}}(M, \mathbb{C})$$

(no. of harmonic (r, s)-forms).

D-branes and Open Strings

Polyakov's action naturally introduces **Neumann b.c.**:

$$\partial_\sigma X(\tau, \sigma)\Big|_{\sigma=0}^{\sigma=\ell} = 0$$

satisfied by **open and closed strings** in D dim. s.t. $\Box X = 0 \Rightarrow X(z, \bar{z}) = X(z) + \overline{X}(\bar{z})$.

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T-duality

Consider **closed strings** on $\mathscr{M}^{1,D-1} = \mathscr{M}^{1,D-2} \otimes \mathrm{S}^1(R)$:

$$\begin{cases} \alpha_0^{D-1} & = \frac{1}{\sqrt{2\alpha'}}\left(n\frac{\alpha'}{R} + mR\right) \\ \widetilde{\alpha}_0^{D-1} & = \frac{1}{\sqrt{2\alpha'}}\left(n\frac{\alpha'}{R} - mR\right) \end{cases} \Rightarrow \quad M^2 = -p^\mu p_\mu = \frac{2}{\alpha'}\left(\alpha_0^{D-1}\right)^2 + \frac{4}{\alpha'}(\mathrm{N} + a)$$

$$= \frac{2}{\alpha'}\left(\widetilde{\alpha}_0^{D-1}\right)^2 + \frac{4}{\alpha'}\left(\widetilde{\mathrm{N}} + a\right)$$

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T-duality

Dirichlet b.c. consequence of **T-duality** on p directions:

$$\overline{X}(z) \mapsto -\overline{X}(z) \quad \Rightarrow \quad \partial_\sigma X^i(\tau,\sigma)\Big|_{\sigma=0}^{\sigma=\ell} = 0 \quad \xrightarrow{T-\text{duality}} \quad \partial_\tau \widetilde{X}^i(\tau,\sigma)\Big|_{\sigma=0}^{\sigma=\ell} = 0$$

thus **open strings** can be **constrained** to $D(D-p-1)$-branes. [Polchinski (1995, 1996)]

D-branes and Open Strings

Introducing Dp-branes breaks $\boxed{\mathrm{ISO}(1,\, D-1) \to \mathrm{ISO}(1, p) \otimes \mathrm{SO}(D-1-p)}$:

$$\mathcal{A}^{\mu} \to \left(\mathcal{A}^{A},\, \mathcal{A}^{a}\right) \quad \Rightarrow \quad \mathrm{U}(1) \text{ theory in } p+1 \text{ dimensions (and scalars)}$$

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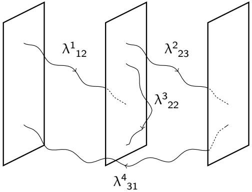

[Chan, Paton (1969)]

$$|n;\, r\rangle = \sum_{i,j=1}^{N} |n;\, i,\, j\rangle\, \lambda^{r}{}_{ij} \quad \Rightarrow \quad \boxed{\mathrm{U}(N)}$$

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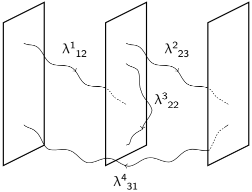

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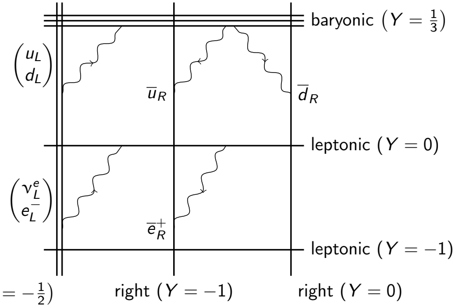

$$|n;\, r\rangle = \sum_{i,j=1}^{N} |n;\, i,\, j\rangle\, \lambda^r{}_{ij} \quad \Rightarrow \quad \boxed{\mathrm{U}(N)}$$

Intersecting D-branes

Consider $\boxed{\text{3 intersecting } D6\text{-branes}}$ filling $\mathscr{M}^{1,3}$ and **embedded in** $\mathbb{R}^6$

Twist Fields Correlators

$$\left\langle \prod_{t=1}^{N_B} \sigma_{\mathrm{M}_{(t)}}\left(x_{(t)}\right) \right\rangle = \mathcal{N}\left(\left\{x_{(t)},\, \mathrm{M}_{(t)}\right\}_{1\leq t\leq N_B}\right) e^{-S_{E\,(\mathbf{cl})}\left(\left\{x_{(t)},\, \mathrm{M}_{(t)}\right\}_{1\leq t\leq N_B}\right)}$$

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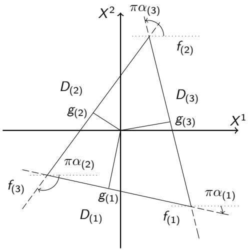
Intersecting D-branes

Consider 3 intersecting $D6$-branes filling $\mathscr{M}^{1,3}$ and **embedded in** $\mathbb{R}^6$

Twist Fields Correlators

$$\left\langle \prod_{t=1}^{N_B} \sigma_{\mathrm{M}_{(t)}}\left(x_{(t)}\right) \right\rangle = \mathcal{N}\left(\left\{x_{(t)},\, \mathrm{M}_{(t)}\right\}_{1\le t\le N_B}\right) e^{-S_{E\,(\mathrm{cl})}\left(\left\{x_{(t)},\, \mathrm{M}_{(t)}\right\}_{1\le t\le N_B}\right)}$$



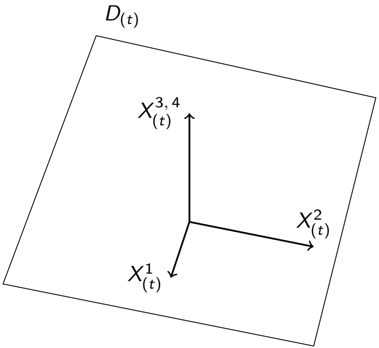
D-branes in **factorised** internal space:

- **embedded as lines** in $\mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2$
- **relative rotations** are $\mathrm{SO}(2) \simeq \mathrm{U}(1)$ elements
- $S_E^{(\mathrm{cl})}\left(\left\{x_{(t)},\, \mathrm{M}_{(t)}\right\}_{1\le t\le N_B}\right) \sim \mathrm{Area}\left(\left\{f_{(t)},\, \mathrm{R}_{(t)}\right\}_{1\le t\le N_B}\right)$

[Cremades, Ibanez, Marchesano (2003); Pesando (2012)]

$\mathrm{SO}(4)$ Rotations

Consider $\boxed{\mathbb{R}^4 \times \mathbb{R}^2}$ (focus on $\mathbb{R}^4$):



$$\left(X_{(t)}\right)^I = \left(R_{(t)}\right)^I{}_J X^J - g^I_{(t)} \in \mathbb{R}^4$$

where

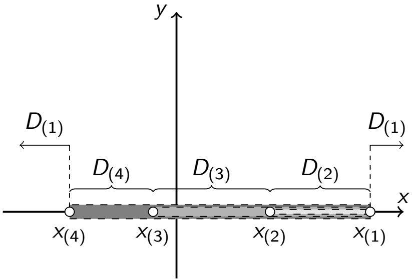
$$R_{(t)} \in \frac{\mathrm{SO}(4)}{\mathrm{S}(\mathrm{O}(2) \times \mathrm{O}(2))}$$

that is

$$\left[R_{(t)}\right] = \left\{R_{(t)} \sim \mathcal{O}_{(t)} R_{(t)}\right\}$$

Boundary Conditions and Open Strings

- $u = x + iy = e^{\tau_e + i\sigma}$ and $\overline{u} = u^*$
- $x_{(t)} < x_{(t-1)}$ **worldsheet intersection points**



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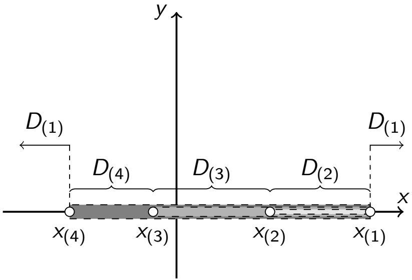

Branch Cuts and Discontinuities for $x \in D_{(t)}$

$$\begin{cases} \partial_u X(x + i0^+) & = U_{(t)} \cdot \partial_{\overline{u}} \overline{X}(x - i0^+) = \left[R_{(t)}^{-1} \cdot (\sigma_3 \otimes \mathbb{1}_2) \cdot R_{(t)} \right] \cdot \partial_{\overline{u}} \overline{X}(x - i0^+) \\ X(x_{(t)}, x_{(t)}) & = f_{(t)} \end{cases}$$

Doubling Trick and Spinor Representation

Doubling Trick

$$\partial_z \mathcal{X}(z) = \begin{cases} \partial_u X(u) & \text{if } z \in \mathscr{H}_>^{(\bar{t})} \\ U_{(\bar{t})}\, \partial_{\bar{u}} \overline{X}(\overline{u}) & \text{if } z \in \mathscr{H}_<^{(\bar{t})} \end{cases} \quad \Rightarrow \quad \begin{aligned} \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i}\delta_+) &= \mathcal{U}_{(t,\, t+1)}\, \partial_z \mathcal{X}(x_{(t)} + \delta_+), \\ \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i}\delta_-) &= \widetilde{\mathcal{U}}_{(t,\, t+1)}\, \partial_z \mathcal{X}(x_{(t)} + \delta_-), \end{aligned}$$

where $\mathscr{H}_{\gtrless}^{(t)} = \left\{ z \in \mathbb{C} \mid \operatorname{Im} z \gtrless 0 \text{ or } z \in D_{(t)} \right\}$ and $\delta_\pm = \eta \pm i0^+$.

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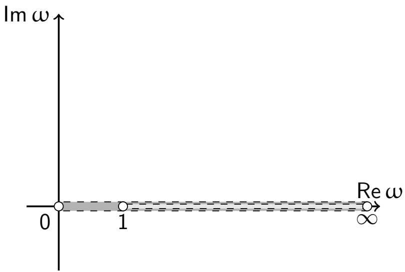
Use Pauli matrices $\tau = (i\,\mathbb{1}_2, \vec{\sigma})$:

$$\partial_z \mathcal{X}_{(s)}(z) = \partial_z \mathcal{X}^I(z)\, \tau_I \quad \Rightarrow \quad \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i}\,\delta_\pm) = \overset{(\sim)}{\mathcal{L}}_{(t,\,t+1)}\, \partial_z \mathcal{X}(x_{(t)} + \delta_\pm)\, \overset{(\sim)}{\mathcal{R}}_{(t,\,t+1)}$$

where

$$\overset{(\sim)}{\mathcal{L}}_{(t,\,t+1)} \in \mathrm{SU}(2)_L \quad \text{and} \quad \overset{(\sim)}{\mathcal{R}}_{(t,\,t+1)} \in \mathrm{SU}(2)_R$$

Hypergeometric Basis

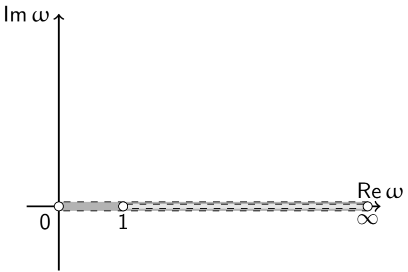


Sum over all contributions:

$$\partial_z \mathcal{X}(z) = \frac{\partial \omega_z}{\partial z} \sum_{l,\, r=-\infty}^{+\infty} c_{lr} \left(-\omega_z\right)^{A_{lr}} \left(1-\omega_z\right)^{B_{lr}}$$

$$\times\, B^{(L)}_{0,\, l}(\omega_z) \left(B^{(R)}_{0,\, r}(\omega_z) \right)^{T}$$

Hypergeometric Basis



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Basis of Solutions

$$B_{0,\, n}(\omega_z) = \begin{pmatrix} 1 & 0 \\ 0 & K_n \end{pmatrix} \begin{pmatrix} \frac{1}{\Gamma(c_n)}\, {}_2F_1\left(a_n,\, b_n;\, c_n;\, \omega_z\right) \\ \frac{(-\omega_z)^{1-c_n}}{\Gamma(2-c_n)}\, {}_2F_1\left(a_n+1-c_n,\, b_n+1-c_n;\, 2-c_n;\, \omega_z\right) \end{pmatrix}$$

The Solution

Sequence of the operations:

1. rotation matrix = monodromy matrix
2. contiguity relations $\Rightarrow$ independent hypergeometrics
3. finite action $\Rightarrow$ 2 solutions (no. of d.o.f. is correctly saturated)
4. boundary conditions $\Rightarrow$ fix free constants c_{lr}

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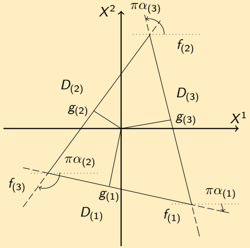
Physical Interpretation



$$2\pi\alpha' \, S_{\mathbb{R}^4}\Big|_{\text{on-shell}} = \sum_{t=1}^{3}\left(\frac{1}{2}\left|g_{(t)}^{\perp}\right|\left|f_{(t-1)} - f_{(t)}\right|\right) = \text{Area}\left(\left\{f_{(t)}\right\}_{1\le t\le N_B}\right)$$

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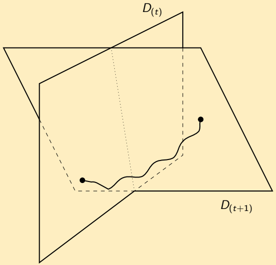
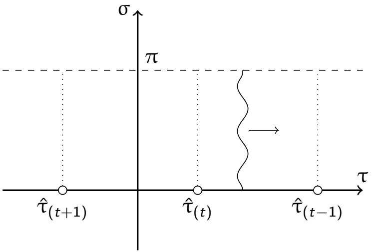


- strings no longer confined to plane
- strings form a *small bump* from the D-brane
- classical action **larger** than factorised case

[RF, Pesando (2019)]

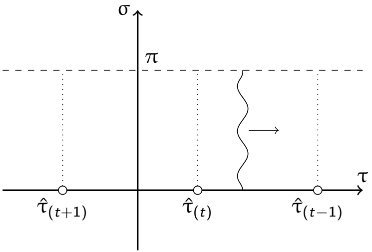
Fermions on the Strip



Action of Boundary Changing Operators

$$
\begin{cases}
\psi^i_-(\tau,0) & = \left(R_{(t)}\right)^I{}_J\,\psi^J_+(\tau,0) \quad \text{for } \tau \in \left(\hat{\tau}_{(t)},\, \hat{\tau}_{(t-1)}\right) \\
\psi^I_-(\tau,\pi) & = -\psi^I_+(\tau,\pi) \quad \text{for } \tau \in \mathbb{R}
\end{cases}
$$

Fermions on the Strip



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$$\mathcal{T}_{\pm\pm}(\xi_{\pm}) = -i\,\frac{T}{4}\,\psi^{*}_{\pm,I}(\xi_{\pm})\,\overset{\leftrightarrow}{\partial}\psi^{I}_{\pm}(\xi_{\pm}) \quad \Rightarrow \quad \begin{cases} \dot{\mathrm{H}}(\tau) & = 0 \\ \dot{\mathrm{P}}(\tau) & \neq 0 \end{cases} \quad \Leftrightarrow \quad \tau \in \left(\tau_{(t)},\, \tau_{(t-1)}\right)$$

Conserved Product and Operators

Expand on a **basis of solutions**

$$\psi_\pm(\xi_\pm) = \sum_{n=-\infty}^{+\infty} b_n \, \psi_n(\xi_\pm) \qquad \Rightarrow \qquad \Psi(z) = \begin{cases} \psi_{E,+}(u) & \text{if } z \in \mathscr{H}_>^{(\bar{t})} \\ \psi_{E,-}(u) & \text{if } z \in \mathscr{H}_<^{(\bar{t})} \end{cases}$$

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$$\langle\!\langle {}^*\psi_n, \psi_m \rangle = 2\pi\mathcal{N} \oint \frac{\mathrm{d}z}{2\pi i} \, {}^*\Psi_n \, {}^*\Psi_m = \delta_{n,m} \qquad \Rightarrow \qquad \left\langle\!\!\left\langle {}^*\Psi_n{}^{(*)}, \Psi^{(*)} \right\rangle = b_n^{(\dagger)}$$

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Derive the **algebra of operators:**

$$\left[b_n, b_m^\dagger\right]_+ = \frac{2\mathcal{N}}{T} \, \langle\!\langle {}^*\Psi_n{}^*, \Psi_m^* \rangle$$

Twisted Complex Fermions

Consider $R_{(t)} = e^{i\pi\alpha_{(t)}} \in \mathrm{U}(1)$:

$$\Psi(x_{(t)} + e^{2\pi i}\delta) = e^{i\pi\epsilon_{(t)}}\Psi(x_{(t)} + \delta)$$

where

$$\epsilon_{(t)} = \alpha_{(t+1)} - \alpha_{(t)} + \theta\left(\alpha_{(t)} - \alpha_{(t+1)} - 1\right) - \theta\left(\alpha_{(t+1)} - \alpha_{(t)} - 1\right)$$

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Basis of Solutions

$$\Psi_n\big(z;\,\{x_{(t)}\}\big) = \mathcal{N}_\Psi\, z^{-n} \prod_{t=1}^{N} \left(1 - \frac{z}{x_{(t)}}\right)^{n_{(t)} + \frac{\epsilon_{(t)}}{2}}$$

$$^{*}\Psi_n\big(z;\,\{x_{(t)}\}\big) = \frac{1}{2\pi\mathcal{N}\mathcal{N}_\Psi}\, z^{n-1} \prod_{t=1}^{N} \left(1 - \frac{z}{x_{(t)}}\right)^{-\widetilde{n}_{(t)} + \frac{\epsilon_{(t)}}{2}}$$

Vacua

Define the **vacuum** with respect to b_n:

$$b_n \left| \left\{ x_{(t)} \right\} \right\rangle = 0 \quad \text{for} \quad n \geq 1$$

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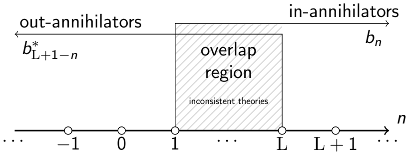

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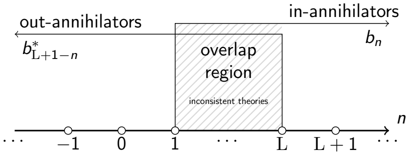

Stress-energy Tensor and CFT Approach

Compute the OPEs leading to the time dependent **stress-energy tensor:**

$$\mathcal{T}(z) = \frac{\pi T}{2} \mathcal{N}_\Psi^2 \sum_{n,\, m=-\infty}^{+\infty} : b_n\, b_m^* : \; z^{-n-m} \left[\frac{m-n}{2} + 2 \sum_{t=1}^{N} \frac{n_{(t)} + \frac{\epsilon_{(t)}}{2}}{z - x_{(t)}} \right] + \frac{1}{2} \left(\sum_{t=1}^{N} \frac{n_{(t)} + \frac{\epsilon_{(t)}}{2}}{z - x_{(t)}} \right)^2$$

[RF, Pesando (2019)]

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[RF, Pesando (2019)]

Invariant Vacuum and Spin Fields

$$\left|\{x_{(t)}\}\right\rangle = \mathcal{N}\left(\{x_{(t)}\}\right) \mathrm{R}\left[\prod_{t=1}^{M} S_{(t)}(x_{(t)}) \right] |0\rangle_{\mathrm{SL}_2(\mathbb{R})}$$

Spin Fields Amplitudes

Equivalence with Bosonization

$$\partial_{x_{(t)}} \ln \left\langle \{x_{(t)}\} \middle| \{x_{(t)}\} \right\rangle = \oint_{x_{(t)}} \frac{\mathrm{d}z}{2\pi i} \frac{\left\langle \{x_{(t)}\} \middle| \mathcal{T}(z) \middle| \{x_{(t)}\} \right\rangle}{\left\langle \{x_{(t)}\} \middle| \{x_{(t)}\} \right\rangle}$$

$$\Rightarrow \quad \left\langle \{x_{(t)}\} \middle| \{x_{(t)}\} \right\rangle = \mathcal{N}\left(\{\epsilon_{(t)}\}\right) \prod_{\substack{t=1 \\ t>u}}^{N} \left(x_{(u)} - x_{(t)}\right)^{\left(n_{(u)} + \frac{\epsilon_{(u)}}{2}\right)\left(n_{(t)} + \frac{\epsilon_{(t)}}{2}\right)}$$

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- (semi-)phenomenological models involve **twist and spin** fields and **open strings**
- framework for **bosonic** open strings with **intersecting D-branes**
- **spin fields** as **boundary changing operators** (hidden in **defects**)
- framework for amplitudes (extension to (non) Abelian twist/spin fields?)

Contents

A Few Words on a Theory of Everything

string theory = theory of everything = **nuclear forces** + **gravity**

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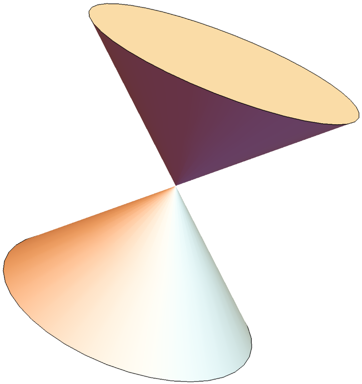

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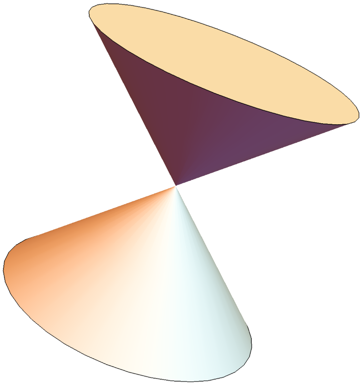

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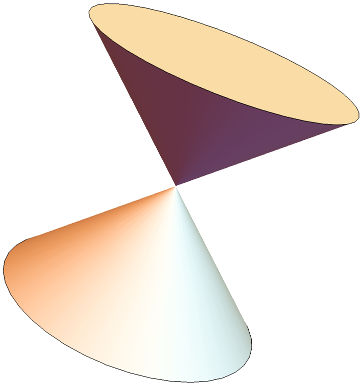

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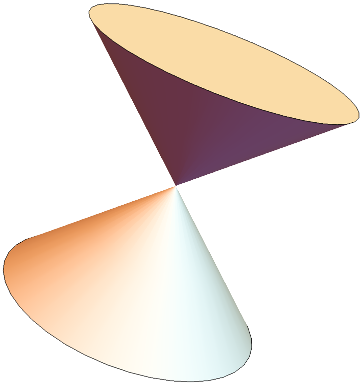

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$\Downarrow$

time-dependent orbifold models

[Craps, Kutasov, Rajesh (2002); Liu, Moore, Seiberg (2002)]

Cosmological Singularities

Use **time-dependent orbifolds** to model **space-like singularities**:

divergent $\boxed{\text{closed string}}$ amplitudes $\Rightarrow$ gravitational backreaction?

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Divergences

Even in simple models (e.g. NBO, more on this later) the 4 tachyons amplitude is divergent **in the open sector at tree level**:

$$A_4 \sim \int_{q \sim \infty} \frac{\mathrm{d}q}{|q|} \mathscr{A}(q)$$

where

$$\mathscr{A}_{\text{closed}}(q) \sim q^{4-\alpha' \|\vec{p}_\perp\|^2} \qquad \text{and} \qquad \mathscr{A}_{\text{open}}(q) \sim q^{1-\alpha' \|\vec{p}_\perp\|^2} \operatorname{tr}([T_1,\, T_2]_+ \, [T_3,\, T_4]_+)$$

Null Boost Orbifold

Start from $\left(x^+,\, x^-,\, x^2,\, \vec{x}\right) \in \mathscr{M}^{1,\, D-1}$:

$$\begin{cases} u &= x^- \\ z &= \frac{x^2}{\Delta\, x^-} \\ v &= x^+ - \frac{1}{2} \frac{\left(x^2\right)^2}{x^-} \end{cases} \qquad \Rightarrow \qquad \mathrm{d}s^2 = -2\, \mathrm{d}u\, \mathrm{d}v + \left(\Delta\, u\right)^2\, \mathrm{d}z^2 + \delta_{ij}\, \mathrm{d}x^i\, \mathrm{d}x^j$$

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$$\kappa = -i(2\pi\Delta)J_{+2} = 2\pi\partial_z \quad \Rightarrow \quad z \sim z + 2\pi n$$

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Scalars on NBO:

$$\phi_{\{k_+,\, l,\, \vec{k},\, r\}}(u,\, v,\, z,\, \vec{x}) = e^{i\left(k_+ v + lz + \vec{k}\cdot\vec{x}\right)}\, \widetilde{\phi}_{\{k_+,\, l,\, \vec{k},\, r\}}(u) = \frac{e^{i\left(k_+ v + lz + \vec{k}\cdot\vec{x}\right)}}{\sqrt{(2\pi)^D\, |2\Delta k_+ u|}}\, e^{-i\frac{l^2}{2\Delta^2 k_+}\frac{1}{u} + i\frac{\|\vec{k}\|^2 + r}{2k_+}u}$$

Scalar QED Interactions

Scalar–photon interactions:

$$S_{\mathrm{sQED}}^{(\mathrm{int})} = \int_{\Omega} \mathrm{d}^D x \sqrt{-g} \left(-i\, e\, g^{\alpha\beta} a_\alpha (\phi^* \partial_\beta \phi - \partial_\beta \phi^* \phi) + e^2 g^{\alpha\beta} a_\alpha a_\beta |\phi|^2 - \frac{g_4}{4} |\phi|^4 \right)$$

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Terms involved:

$$\mathcal{I}_{\{N\}}^{[\nu]} = \int_{-\infty}^{+\infty} \mathrm{d}u\, |\Delta\, u| u^\nu \prod_{i=1}^{N} \widetilde{\phi}_{\left\{k_{+\,(i)},\, l_{(i)},\, \vec{k}_{(i)},\, r_{(i)}\right\}}(u)$$

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*most terms **do not converge** due to **isolated zeros** ($l_{(*)} \equiv 0$) and cannot be recovered even with a **distributional interpretion** due to the term $\propto u^{-1}$ in the exponential*

String and Field Theory

So far:

- field theory presents **divergences** (see sQED)
- obvious ways to regularise (Wilson lines, higher derivative couplings, etc.) **do not work**
- divergences are **not (only) gravitational**
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Massive String States

$$V_M(x;\, k,\, S,\, \xi) =: \left(\frac{i}{\sqrt{2\alpha'}} \xi_\alpha \partial_x^2 X^\alpha(x,\, x) + \left(\frac{i}{\sqrt{2\alpha'}} \right)^2 S_{\alpha\beta} \partial_x X^\alpha(x,\, x) \partial_x X^\beta(x,\, x) \right) e^{ik\cdot X(x,\, x)} :$$

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string theory cannot do ***better than field theory*** *(EFT) if the latter* ***does not exist***

Resolution and Motivation

Introduce the generalised NBO:

$$\begin{cases} u &= x^- \\ z &= \frac{1}{2x^-}\left(\frac{x^2}{\Delta_2}+\frac{x^3}{\Delta_3}\right) \\ w &= \frac{1}{2x^-}\left(\frac{x^2}{\Delta_2}-\frac{x^3}{\Delta_3}\right) \\ v &= x^+ - \frac{1}{2x^-}\left(\left(x^2\right)^2+\left(x^3\right)^2\right) \end{cases} \quad \Rightarrow \quad \kappa = -2\pi i(\Delta_2 J_{+2}+\Delta_3 J_{+3}) = 2\pi\partial_z$$

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No isolated zeros $\Rightarrow$ distributional Interpretation

$$\widetilde{\phi}_{\{k_+,\,p,\,l,\,\vec{k},\,r\}}(u) = \frac{1}{2\sqrt{(2\pi)^D|\Delta_2\Delta_3 k_+|}}\frac{1}{|u|}e^{-i\left(\frac{1}{8k_+u}\left[\frac{(l+p)^2}{\Delta_2^2}+\frac{(l-p)^2}{\Delta_3^2}\right]-\frac{\|\vec{k}\|^2+r}{2k_+}u\right)}$$

On the Divergences and Their Nature

- divergences are present in sQED and **open string** sector
- singularities $\Rightarrow$ **massive states** are no longer spectators
- vanishing volume (**compact orbifold directions**) $\Rightarrow$ particles “cannot escape”
- **non compact** orbifold directions $\Rightarrow$ interpretation of **amplitudes as distributions**
- issue not restricted to NBO/GNBO but also BO, null brane, etc. (it is a **general issue** connected to the geometry of the underlying space)

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*divergences are **hidden into EFT contact terms** and interactions with **string massive states**: gravity is not the only cause as the same problems are present also in gauge theories.*

[Arduino, RF, Pesando (2020)]

Contents

The Simplest Calabi–Yau

Focus on Calabi–Yau 3-folds:

$$h^{r,\,s} = \dim_{\mathbb{C}} H^{r,\,s}_{\bar{\partial}}(M,\,\mathbb{C}) \qquad \Rightarrow \qquad \begin{cases} h^{0,\,0} & = h^{3,\,0} = 1 \\ h^{r,\,0} & = 0 \quad \text{if} \quad r \neq 3 \\ h^{r,\,s} & = h^{3-r,\,3-s} \\ h^{1,\,1},\; h^{2,\,1} \in \mathbb{N} & \end{cases}$$

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Complete Intersection Calabi–Yau Manifolds

Intersection of hypersurfaces in

$$\mathcal{A} = \mathbb{P}^{n_1} \times \cdots \times \mathbb{P}^{n_m}$$

where

$$\mathbb{P}^n: \qquad \begin{cases} p_i\left(Z^0,\,\ldots,\,Z^n\right) & = P_{i_1 \ldots i_i} Z^{i_1} \ldots Z^{i_i} = 0 \\ p_i\left(\lambda Z^0,\,\ldots,\,\lambda Z^n\right) & = \lambda^i p_i\left(Z^0,\,\ldots,\,Z^n\right) \end{cases}$$

[Green, Hübsch (1987); Hübsch (1992)]

Representation of the Output

CICY can be generalised to m projective spaces and k equations. The problem is thus mapped to:

$$\begin{array}{cccc} \mathscr{R}: & \mathbb{N}^{m\times k} & \longrightarrow & \mathbb{N} \\ & \left[\begin{array}{c|ccc} \mathbb{P}^{n_1} & a^1_1 & \dots & a^1_k \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{P}^{n_m} & a^m_1 & \dots & a^m_k \end{array}\right] & \longrightarrow & h^{1,1} \quad \text{or} \quad h^{2,1} \end{array}$$

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Machine Learning Approach

What is $\mathscr{R}$ in **machine learning** approach?

$$\mathscr{R}(M) \to \mathscr{R}_n(M;\, w) \to \widehat{h}^{p,q} \qquad \text{s.t.} \qquad \exists n > M > 0 \quad \Big| \quad \mathcal{L}_n\left(\widehat{h}^{p,q},\, h^{p,q}\right) < \epsilon \quad \forall \epsilon > 0$$

Machine Learning

- **optimisation problem** $\Rightarrow$ gradient descent (or similar)

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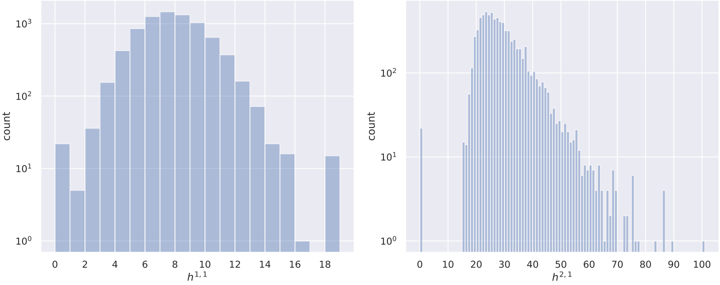
- **optimisation problem** $\Rightarrow$ gradient descent (or similar)
- use **various algorithms** and exploit **large datasets** (more training)
- intersection of **computer science, mathematics and physics**
- provide in-depth **data analysis** of the datasets



[Green *et al.* (1987)]

Dataset

- 7890 CICY manifolds (full dataset)
- **dataset pruning**: no product spaces, no "very far" outliers (reduction of 0.49%)
- $h^{1,1} \in [1, 16]$ and $h^{2,1} \in [15, 86]$
- 80% training, 10% validation, 10% test
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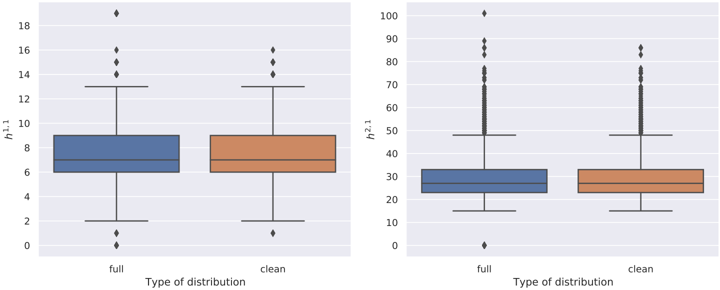

Exploratory Data Analysis

exploratory data analysis $\rightarrow$ feature **selection** $\rightarrow$ Hodge numbers

[Ruehle (2020); Erbin, RF (2020)]

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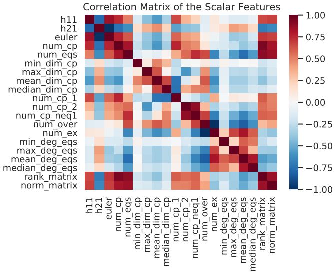
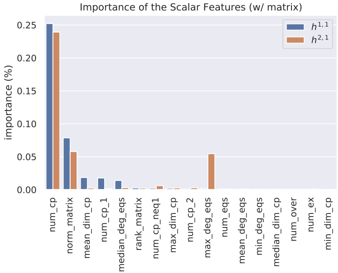
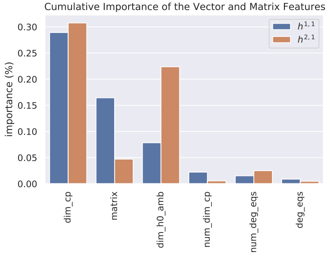
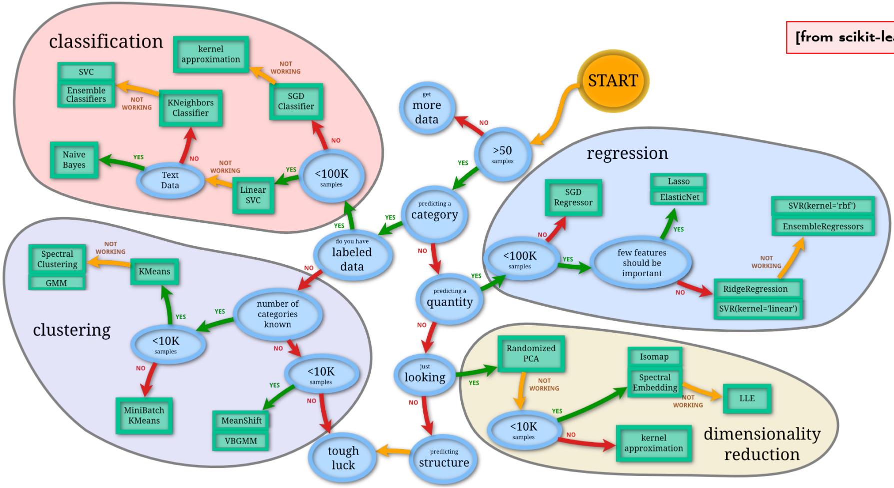






Machine Learning



[from scikit-learn.org]

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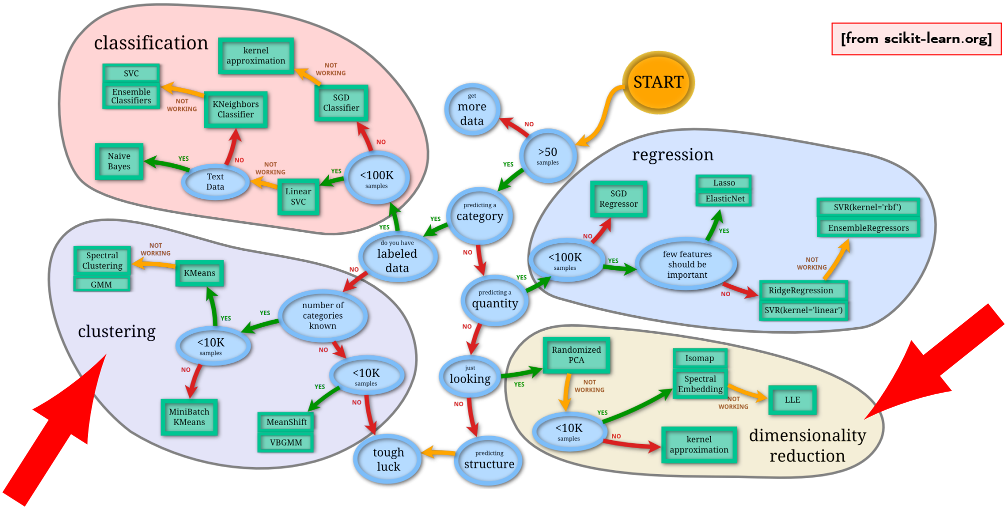

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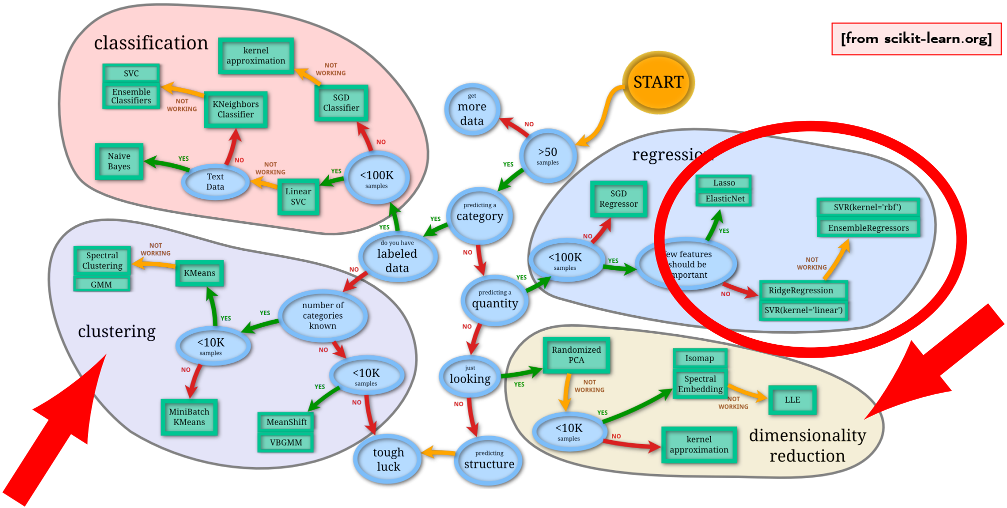

A Word on PCA

What is PCA for a $X \in \mathbb{R}^{n \times p}$?

- project data such that **variance is maximised**
- **eigenvectors** of XX^T or the **singular values** of X
- isolate **signal** from **background**
- ease the ML job of finding a better representation of the input

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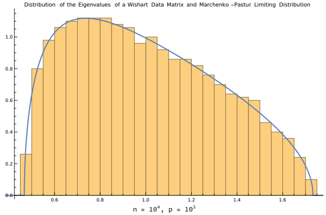
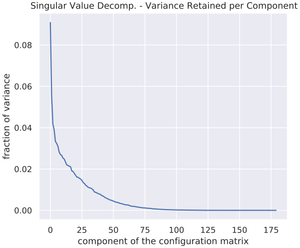
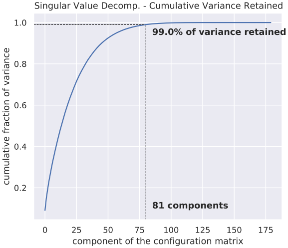
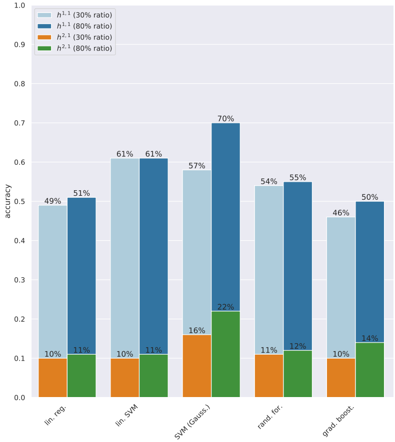






Machine Learning Results

Configuration Matrix Only



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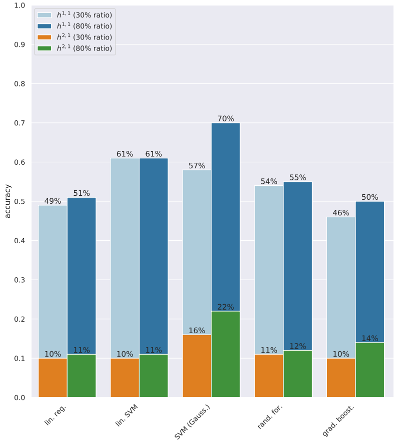
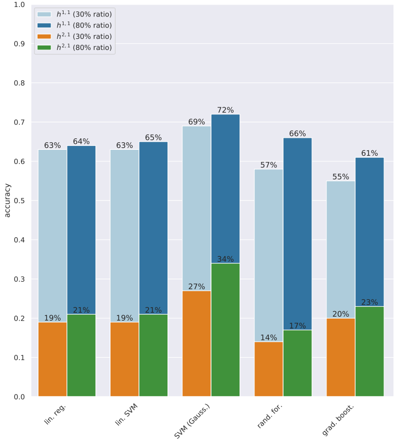


Best Training Set [Erbin, RF (2020)]



Artificial Intelligence and Neural Networks

- use **gradient descent** to optimise **weights**
- learn highly **non linear** representations of the input
- can be "large" to have enough parameters
- can be "deep" to to learn **complicated functions**

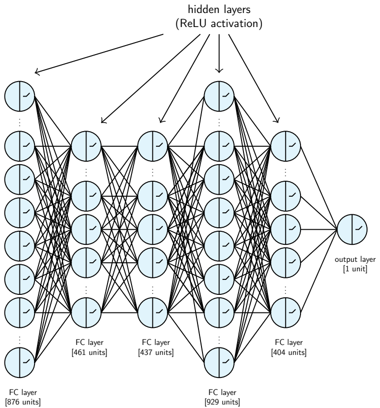
Layers

fully connected: $\phi\left(a^{(i)\,\{l\}} \cdot W^{\{l\}} + b^{\{l\}}\mathbb{1}\right)$

convolutional: $\phi\left(a^{(i)\,\{l\}} * W^{\{l\}} + b^{\{l\}}\mathbb{1}\right)$

Non linearity ensured by:

$$\phi(z) = \mathrm{ReLU}(z) = \max\left(0,\, z\right)$$

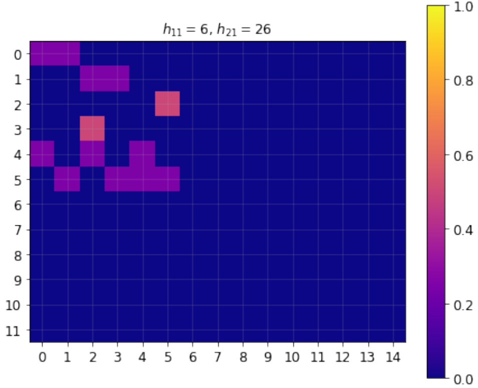


[rendition of the neural network in Bull et al. (2018)]

Convolutional Neural Networks

Why convolutional?

- retain **spacial awareness**
- smaller **no. of parameters** ($\approx 2 \times 10^5$ vs. $\approx 2 \times 10^6$)
- weights are **shared**
- CNNs isolate **"defining features"**
- find patterns as in **computer vision**



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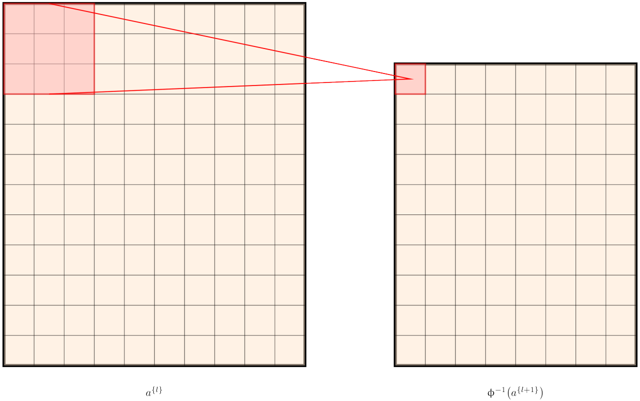

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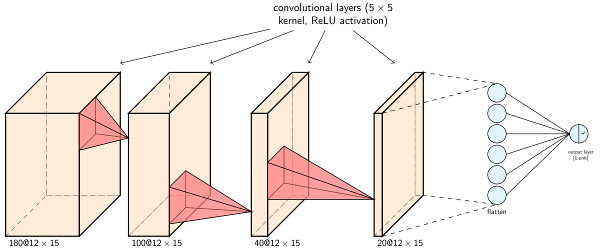

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Recent development by deep learning research at Google led to:

- neural networks with better **generalisation properties**
- **smaller** networks (both parameters and depth)
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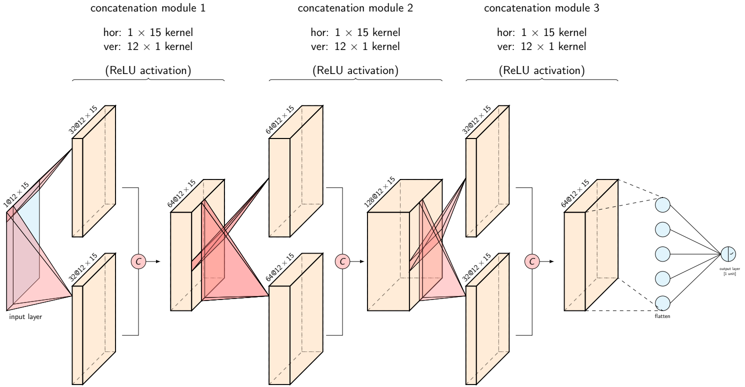
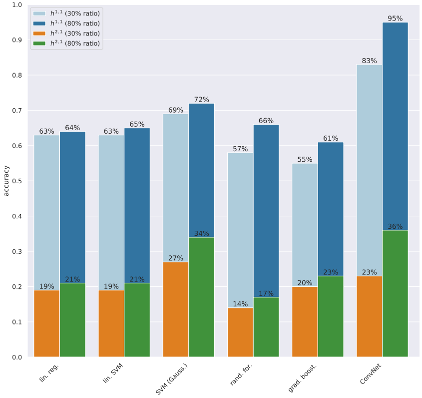


Deep Learning Topology with Computer Vision

Best Training Set [Erbin, RF (2020)]



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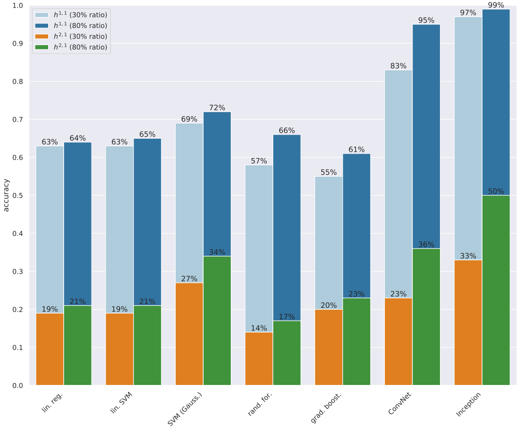

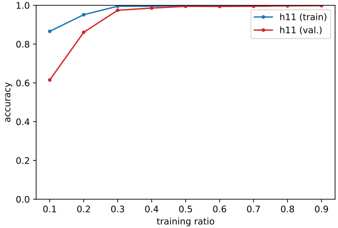

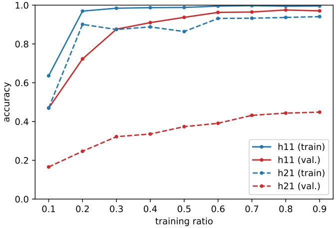

[see Erbin's talk at *string_data 2020*]

A Few Comments and Future Directions

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What now?

- representation learning $\Rightarrow$ what is the best way to represent CICYs?
- study invariances $\Rightarrow$ invariances should not influence the result (graph representations?)
- higher dimensions $\Rightarrow$ what about CICY 4-folds?
- geometric deep learning $\Rightarrow$ explain the geometry of the "AI" behind deep learning!
- reinforcement learning $\Rightarrow$ give the rules, not the result!

The End?

- general framework for **D-branes at angles**
- alternative computations of **correlators of spin fields**
- strings and divergences in **time dependent orbifolds**
- string compactifications and **deep learning techniques**

THANK YOU