D-branes and Deep Learning

Theoretical and Computational Aspects in String Theory

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Action Principle and Conformal Symmetry

Polyakov's Action

$$S_P[\gamma, X, \psi] = -\frac{1}{4\pi} \int_{-\infty}^{+\infty} \mathrm{d}\tau \int_0^{\ell} \mathrm{d}\sigma \sqrt{-\det\gamma}\, \gamma^{\alpha\beta} \left(\frac{2}{\alpha'} \partial_\alpha X^\mu \partial_\beta X^\nu + \psi^\mu \rho_\alpha \partial_\beta \psi^\nu \right) \eta_{\mu\nu}$$

Symmetries:

- **Poincaré transf.** $X'^\mu = \Lambda^\mu{}_\nu X^\nu + c^\mu$
- **2D diff.** $\gamma'_{\alpha\beta} = \left(J^{-1}\right)_{\alpha\beta}{}^{\lambda\rho} \gamma_{\lambda\rho}$
- **Weyl transf.** $\gamma'_{\alpha\beta} = e^{2\omega} \gamma_{\alpha\beta}$

Conformal symmetry:

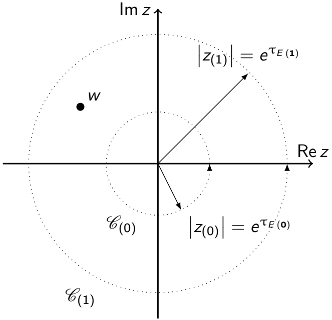
- **vanishing** stress-energy tensor: $\mathcal{T}_{\alpha\beta} = 0$
- **traceless** stress-energy tensor: $\operatorname{tr}\mathcal{T} = 0$
- **conformal gauge** $\gamma_{\alpha\beta} = e^{\phi} \eta_{\alpha\beta}$

Action Principle and Conformal Symmetry

Let $z = e^{\tau_E + i\sigma} \Rightarrow \bar{\partial}\mathcal{T}(z) = \partial\overline{\mathcal{T}}(\bar{z}) = 0$:

$$\mathcal{T}(z)\,\Phi_h(w) \overset{z\to w}{\sim} \frac{h}{(z-w)^2}\Phi_h(w) + \frac{1}{z-w}\partial_w\Phi_h(w)$$

$$\mathcal{T}(z)\,\mathcal{T}(w) \overset{z\to w}{\sim} \frac{\frac{c}{2}}{(z-w)^4} + \mathcal{O}\left((z-w)^{-2}\right)$$



Virasoro algebra $\mathscr{V} \oplus \overline{\mathscr{V}}$

$$\begin{aligned} [L_n, L_m] &= (n-m)L_{n+m} + \frac{c}{12}n\left(n^2-1\right)\delta_{n+m,0} \\ \left[L_n, \bar{L}_m\right] &= 0 \end{aligned}$$

Action Principle and Conformal Symmetry

Superstrings in D dimensions:

$$\mathcal{T}(z) = -\frac{1}{\alpha'}\partial X(z)\cdot\partial X(z) - \frac{1}{2}\psi(z)\cdot\partial\psi(z) \quad\Rightarrow\quad c = \frac{3}{2}D$$

$(\lambda, 0)$ / $(1-\lambda, 0)$ Ghost System

Introduce anti-commuting $(b,\ c)$ and commuting $(\beta,\ \gamma)$ conformal fields:

$$S_{\text{ghost}}[b,\ c,\ \beta,\ \gamma] = \frac{1}{2\pi}\iint \mathrm{d}z\,\mathrm{d}\bar{z}\,\left(b(z)\,\bar{\partial}c(z) + \beta(z)\,\bar{\partial}\gamma(z)\right)$$

where $\lambda_b = 2$ and $\lambda_\beta = \frac{3}{2}$.

Consequence:

$$c_{\text{full}} = c + c_{\text{ghost}} = 0 \quad\Leftrightarrow\quad D = 10.$$

Extra Dimensions and Compactification

BBB

b

CCC

C