D-branes and Deep Learning

Theoretical and Computational Aspects in String Theory

Riccardo Finotello

Scuola di Dottorato in Fisica e Astrofisica

Università degli Studi di Torino
and
I.N.F.N. – sezione di Torino

15th December 2020

Contents

Action Principle and Conformal Symmetry

Polyakov's Action

$$S_P[\gamma, X, \psi] = -\frac{1}{4\pi} \int_{-\infty}^{+\infty} \mathrm{d}\tau \int_0^{\ell} \mathrm{d}\sigma \sqrt{-\det \gamma}\, \gamma^{\alpha\beta} \left(\frac{2}{\alpha'} \partial_\alpha X^\mu \partial_\beta X^\nu + \psi^\mu \rho_\alpha \partial_\beta \psi^\nu \right) \eta_{\mu\nu}$$

Action Principle and Conformal Symmetry

Polyakov's Action

$$S_P[\gamma, X, \psi] = -\frac{1}{4\pi} \int_{-\infty}^{+\infty} \mathrm{d}\tau \int_0^{\ell} \mathrm{d}\sigma \sqrt{-\det\gamma}\,\gamma^{\alpha\beta} \left(\frac{2}{\alpha'} \partial_\alpha X^\mu \partial_\beta X^\nu + \psi^\mu \rho_\alpha \partial_\beta \psi^\nu \right) \eta_{\mu\nu}$$

Symmetries:

- **Poincaré transf.** $X'^\mu = \Lambda^\mu{}_\nu X^\nu + c^\mu$
- **2D diff.** $\gamma'_{\alpha\beta} = \left(\mathrm{J}^{-1}\right)_{\alpha\beta}{}^{\lambda\rho} \gamma_{\lambda\rho}$
- **Weyl transf.** $\gamma'_{\alpha\beta} = e^{2\omega} \gamma_{\alpha\beta}$

Action Principle and Conformal Symmetry

Polyakov's Action

$$S_P[\gamma, X, \psi] = -\frac{1}{4\pi} \int_{-\infty}^{+\infty} \mathrm{d}\tau \int_0^{\ell} \mathrm{d}\sigma \sqrt{-\det \gamma}\, \gamma^{\alpha\beta} \left(\frac{2}{\alpha'} \partial_\alpha X^\mu \partial_\beta X^\nu + \psi^\mu \rho_\alpha \partial_\beta \psi^\nu \right) \eta_{\mu\nu}$$

Symmetries:

- **Poincaré transf.** $X'^\mu = \Lambda^\mu{}_\nu X^\nu + c^\mu$
- **2D diff.** $\gamma'_{\alpha\beta} = \left(\mathrm{J}^{-1}\right)_{\alpha\beta}{}^{\lambda\rho} \gamma_{\lambda\rho}$
- **Weyl transf.** $\gamma'_{\alpha\beta} = e^{2\omega} \gamma_{\alpha\beta}$

Conformal symmetry:

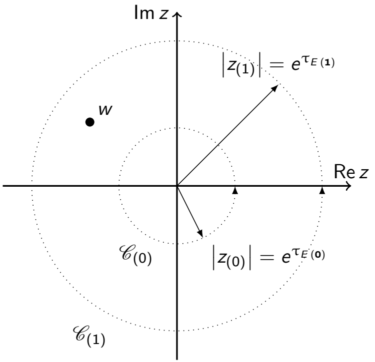
- **vanishing** stress-energy tensor: $\mathcal{T}_{\alpha\beta} = 0$
- **traceless** stress-energy tensor: $\mathrm{tr}\, \mathcal{T} = 0$
- **conformal gauge** $\gamma_{\alpha\beta} = e^{\phi} \eta_{\alpha\beta}$

Action Principle and Conformal Symmetry

Let $z = e^{\tau_E + i\sigma} \Rightarrow \bar{\partial}\mathcal{T}(z) = \partial\overline{\mathcal{T}}(\bar{z}) = 0$:

$$\mathcal{T}(z)\,\Phi_h(w) \overset{z\to w}{\sim} \frac{h}{(z-w)^2}\Phi_h(w) + \frac{1}{z-w}\partial_w\Phi_h(w)$$

$$\mathcal{T}(z)\,\mathcal{T}(w) \overset{z\to w}{\sim} \frac{\frac{c}{2}}{(z-w)^4} + \mathcal{O}\big((z-w)^{-2}\big)$$



Virasoro algebra $\mathscr{V} \oplus \overline{\mathscr{V}}$

$$\begin{aligned} \left[\overset{(-)}{L}_n, \overset{(-)}{L}_m\right] &= (n-m)\overset{(-)}{L}_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n+m,0} \\ \left[L_n, \overline{L}_m\right] &= 0 \end{aligned}$$

Action Principle and Conformal Symmetry

Superstrings in D dimensions:

$$\mathcal{T}(z) = -\frac{1}{\alpha'}\partial X(z)\cdot\partial X(z) - \frac{1}{2}\psi(z)\cdot\partial\psi(z) \quad \Rightarrow \quad c = \frac{3}{2}D$$

Action Principle and Conformal Symmetry

Superstrings in D dimensions:

$$\mathcal{T}(z) = -\frac{1}{\alpha'}\partial X(z)\cdot\partial X(z) - \frac{1}{2}\psi(z)\cdot\partial\psi(z) \quad \Rightarrow \quad c = \frac{3}{2}D$$

$(\lambda, 0)$ / $(1-\lambda, 0)$ Ghost System

Introduce anti-commuting $(b,\ c)$ and commuting $(\beta,\ \gamma)$ conformal fields:

$$S_{\text{ghost}}[b,\ c,\ \beta,\ \gamma] = \frac{1}{2\pi}\iint \mathrm{d}z\,\mathrm{d}\bar{z}\,\left(b(z)\,\bar{\partial}c(z) + \beta(z)\,\bar{\partial}\gamma(z)\right)$$

where $\lambda_b = 2$ and $\lambda_c = -1$, and $\lambda_\beta = \frac{3}{2}$ and $\lambda_\gamma = -\frac{1}{2}$.

Action Principle and Conformal Symmetry

Superstrings in D dimensions:

$$\mathcal{T}(z) = -\frac{1}{\alpha'}\partial X(z) \cdot \partial X(z) - \frac{1}{2}\psi(z) \cdot \partial\psi(z) \quad \Rightarrow \quad c = \frac{3}{2}D$$

$(\lambda, 0)$ / $(1-\lambda, 0)$ Ghost System

Introduce anti-commuting $(b,\ c)$ and commuting $(\beta,\ \gamma)$ conformal fields:

$$S_{\text{ghost}}[b,\ c,\ \beta,\ \gamma] = \frac{1}{2\pi}\iint \mathrm{d}z\, \mathrm{d}\bar{z}\, \left(b(z)\,\overline{\partial}c(z) + \beta(z)\,\overline{\partial}\gamma(z)\right)$$

where $\lambda_b = 2$ and $\lambda_c = -1$, and $\lambda_\beta = \frac{3}{2}$ and $\lambda_\gamma = -\frac{1}{2}$.

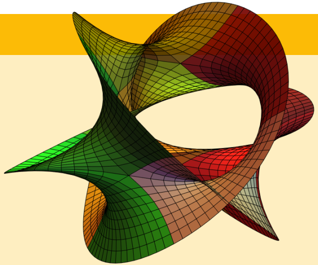
Consequence:

$$c_{\text{full}} = c + c_{\text{ghost}} = 0 \quad \Leftrightarrow \quad D = 10.$$

Extra Dimensions and Compactification

Compactification

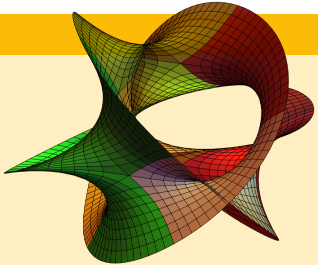
$$\mathscr{M}^{1,9} = \mathscr{M}^{1,3} \otimes \mathscr{X}_6$$



- $\mathscr{X}_6$ is a **compact** manifold
- $N = 1$ **supersymmetry** is preserved in 4D
- algebra of $\mathrm{SU}(3) \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1)$ in arising **gauge group**

Extra Dimensions and Compactification

Compactification



$$\mathscr{M}^{1,9} = \mathscr{M}^{1,3} \otimes \mathscr{X}_6$$

- $\mathscr{X}_6$ is a **compact** manifold
- $N = 1$ **supersymmetry** is preserved in 4D
- algebra of $\mathrm{SU}(3) \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1)$ in arising **gauge group**

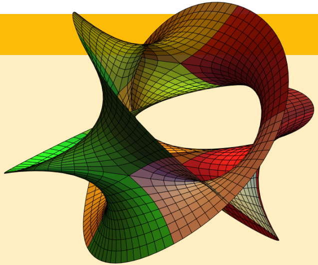
Kähler manifolds (M, g) such that

- $\dim_{\mathbb{C}} M = m$
- $\mathrm{Hol}(g) \subseteq \mathrm{SU}(m)$
- g is Ricci-flat (equiv. $c_1(M)$ vanishes)

Extra Dimensions and Compactification

Compactification

$$\mathscr{M}^{1,9} = \mathscr{M}^{1,3} \otimes \mathscr{X}_6$$



- $\mathscr{X}_6$ is a **compact** manifold
- $N = 1$ **supersymmetry** is preserved in 4D
- algebra of $\mathrm{SU}(3) \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1)$ in arising **gauge group**

Kähler manifolds (M, g) such that

- $\dim_{\mathbb{C}} M = m$
- $\mathrm{Hol}(g) \subseteq \mathrm{SU}(m)$
- g is Ricci-flat (equiv. $c_1(M)$ vanishes)

Characterised by Hodge numbers

$$h^{r,s} = \dim_{\mathbb{C}} H^{r,s}_{\bar{\partial}}(M, \mathbb{C})$$

counting the no. of harmonic (r, s)-forms.

D-branes and Open Strings

Polyakov's action naturally introduces Neumann b.c.:

$$\partial_\sigma X(\tau,\sigma)\Big|_{\sigma=0}^{\sigma=\ell} = 0$$

satisfied by **open and closed** strings living in D dimensions s.t. $\Box X = 0$.

D-branes and Open Strings

Polyakov's action naturally introduces Neumann b.c.:

$$\partial_\sigma X(\tau, \sigma)\Big|_{\sigma=0}^{\sigma=\ell} = 0$$

satisfied by **open and closed** strings living in D dimensions s.t. $\square X = 0$.

T-duality

$$X(z, \bar{z}) = X(z) + \overline{X}(\bar{z}) \quad \overset{T}{\Rightarrow} \quad X(z) - \overline{X}(\bar{z}) = Y(z, \bar{z}) = Y(z) + \overline{Y}(\bar{z})$$

D-branes and Open Strings

Polyakov's action naturally introduces Neumann b.c.:

$$\partial_\sigma X(\tau,\sigma)\Big|_{\sigma=0}^{\sigma=\ell} = 0$$

satisfied by **open and closed** strings living in D dimensions s.t. $\Box X = 0$.

T-duality

$$X(z,\bar{z}) = X(z) + \overline{X}(\bar{z}) \quad \overset{T}{\Rightarrow} \quad X(z) - \overline{X}(\bar{z}) = Y(z,\bar{z}) = Y(z) + \overline{Y}(\bar{z})$$

Resulting effect (repeated $p \leq D-1$ times) leads to Dirichlet b.c.:

$$\partial_\sigma X^i(\tau,\sigma)\Big|_{\sigma=0}^{\sigma=\ell} = 0 \quad \overset{T}{\Rightarrow} \quad \partial_\tau Y^i(\tau,\sigma)\Big|_{\sigma=0}^{\sigma=\ell} = 0 \quad \forall i = 1, 2, \ldots, p$$

thus **open strings** can be **constrained** to $D(D-p-1)$-branes.

D-branes and Open Strings

Introducing *Dp*-branes breaks $\boxed{\mathrm{ISO}(1,\ D-1) \to \mathrm{ISO}(1,p) \otimes \mathrm{SO}(D-1-p).}$

D-branes and Open Strings

Introducing Dp-branes breaks $\boxed{\mathrm{ISO}(1,\, D-1) \to \mathrm{ISO}(1,p) \otimes \mathrm{SO}(D-1-p).}$

Massless Spectrum (*irrep* of little group $\mathrm{SO}(D-2)$)

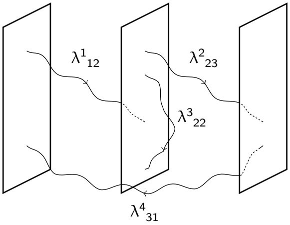
$$\mathcal{A}^{\mu} \quad \leftrightarrow \quad \alpha^{\mu}_{-1}\left|0\right\rangle \qquad \longrightarrow \qquad \begin{array}{llll} \mathcal{A}^{A} & \leftrightarrow & \alpha^{A}_{-1}\left|0\right\rangle, & A = 0,\, 1,\, \dots,\, p \\ \mathcal{A}^{a} & \leftrightarrow & \alpha^{a}_{-1}\left|0\right\rangle, & a = 1,\, 2,\, \dots,\, D-p-1 \end{array}$$

D-branes and Open Strings

Introducing Dp-branes breaks $\boxed{\mathrm{ISO}(1,\, D-1) \to \mathrm{ISO}(1, p) \otimes \mathrm{SO}(D-1-p).}$

Massless Spectrum (*irrep* of little group $\mathrm{SO}(D-2)$)

$$\mathcal{A}^{\mu} \quad \leftrightarrow \quad \alpha^{\mu}_{-1}\,|0\rangle \quad \longrightarrow \quad \begin{array}{lll} \mathcal{A}^{A} \quad \leftrightarrow & \alpha^{A}_{-1}\,|0\rangle\,, & A = 0,\, 1,\, \dots,\, p \\ \mathcal{A}^{a} \quad \leftrightarrow & \alpha^{a}_{-1}\,|0\rangle\,, & a = 1,\, 2,\, \dots,\, D-p-1 \end{array}$$



Introducing **Chan–Paton factors** $\lambda^r{}_{ij}$, when branes are **coincident**:

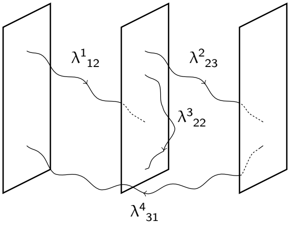
$$\bigoplus_{r=1}^{N} \mathrm{U}_r(1) \quad \longrightarrow \quad \mathrm{U}(N)$$

D-branes and Open Strings

Introducing Dp-branes breaks $\boxed{\mathrm{ISO}(1,\, D-1) \to \mathrm{ISO}(1, p) \otimes \mathrm{SO}(D-1-p).}$

Massless Spectrum (*irrep* of little group $\mathrm{SO}(D-2)$)

$$\mathcal{A}^{\mu} \quad \leftrightarrow \quad \alpha^{\mu}_{-1} \left|0\right\rangle \qquad \longrightarrow \qquad \begin{array}{lll} \mathcal{A}^{A} & \leftrightarrow \quad \alpha^{A}_{-1} \left|0\right\rangle, & A = 0, 1, \dots, p \\ \mathcal{A}^{a} & \leftrightarrow \quad \alpha^{a}_{-1} \left|0\right\rangle, & a = 1, 2, \dots, D-p-1 \end{array}$$

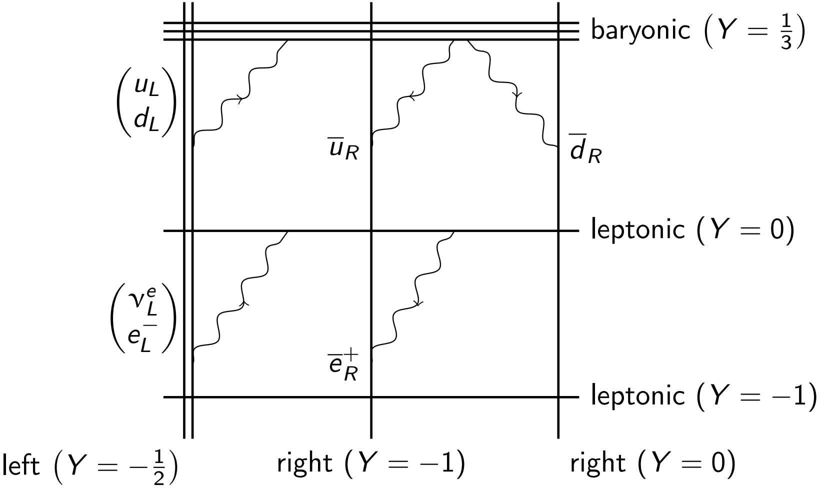


Introducing **Chan–Paton factors** $\lambda^{r}{}_{ij}$, when branes are **coincident**:

$$\bigoplus_{r=1}^{N} \mathrm{U}_r(1) \quad \longrightarrow \quad \mathrm{U}(N)$$

Build gauge bosons, fermions and scalars.

Standard Model-like Scenarios



Intersecting D-branes

Consider N intersecting $D6$-branes filling $\mathscr{M}^{1,3}$ and **embedded in** $\mathbb{R}^6$

Twist Fields Correlators

$$\left\langle \prod_{t=1}^{N} \sigma_{\mathrm{M}_{(t)}}\left(x_{(t)}\right) \right\rangle = \mathcal{N}\left(\left\{x_{(t)},\, \mathrm{M}_{(t)}\right\}_{1\leq t\leq N_B}\right) e^{-S_{E\,(\mathbf{cl})}\left(\left\{x_{(t)},\,\mathrm{M}_{(t)}\right\}_{1\leq t\leq N_B}\right)}$$

Intersecting D-branes

Consider N intersecting $D6$-branes filling $\mathscr{M}^{1,3}$ and **embedded in** $\mathbb{R}^6$

Twist Fields Correlators

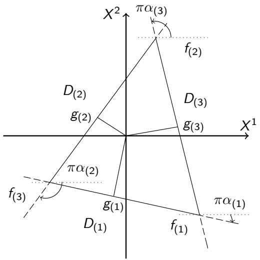
$$\left\langle \prod_{t=1}^{N} \sigma_{\mathrm{M}_{(t)}}\left(x_{(t)}\right) \right\rangle = \mathcal{N}\left(\left\{ x_{(t)}, \mathrm{M}_{(t)} \right\}_{1 \le t \le N_B} \right) e^{-S_{E\,(\mathbf{cl})}\left(\left\{ x_{(t)}, \mathrm{M}_{(t)} \right\}_{1 \le t \le N_B} \right)}$$

Intersecting D-branes

Consider N intersecting $D6$-branes filling $\mathscr{M}^{1,3}$ and **embedded in** $\mathbb{R}^6$

Twist Fields Correlators

$$\left\langle \prod_{t=1}^{N} \sigma_{\mathrm{M}_{(t)}}\left(x_{(t)}\right) \right\rangle = \mathcal{N}\left(\left\{ x_{(t)},\, \mathrm{M}_{(t)} \right\}_{1 \le t \le N_B} \right) e^{-S_{E\,(\mathbf{cl})}\left(\left\{ x_{(t)},\, \mathrm{M}_{(t)} \right\}_{1 \le t \le N_B} \right)}$$



D-branes in **factorised** internal space:

- **embedded as lines** in $\mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2$
- **relative rotations** are $\mathrm{SO}(2) \simeq \mathrm{U}(1)$ elements
- $S_{E\,(\mathrm{cl})}\left(\left\{ x_{(t)},\, \mathrm{M}_{(t)} \right\}_{1 \le t \le N_B} \right) \sim \mathrm{Area}\left(\left\{ f_{(t)},\, \mathrm{R}_{(t)} \right\}_{1 \le t \le N_B} \right)$

BBB

b

CCC

C