

D-branes and Deep Learning

Theoretical and Computational Aspects in String Theory

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15th December 2020



Contents

Conformal Symmetry and Geometry of the Worldsheet

Preliminary Concepts and Tools

D-branes Intersecting at Angles

Fermions and Point-like Defect CFT

Cosmological Backgrounds and Divergences

Orbifolds and Cosmological Toy Models

Null Boost Orbifold

Deep Learning the Geometry of String Theory

Machine Learning and Deep Learning

Machine Learning for String Theory

AI Implementations for Geometry and Strings

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Action Principle and Conformal Symmetry

Polyakov's Action

$$S_P[\gamma, X, \psi] = -\frac{1}{4\pi} \int_{-\infty}^{+\infty} d\tau \int_0^\ell d\sigma \sqrt{-\det \gamma} \gamma^{\alpha\beta} \left(\frac{2}{\alpha'} \partial_\alpha X^\mu \partial_\beta X^\nu + \psi^\mu \rho_\alpha \partial_\beta \psi^\nu \right) \eta_{\mu\nu}$$

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Symmetries:

Poincaré transf.: $X'^\mu = \Lambda^\mu_\nu X^\nu + c^\mu$
2D diff.: $\gamma'_{\alpha\beta} = (J^{-1})_{\alpha\beta}^{\lambda\rho} \gamma_{\lambda\rho}$
Weyl transf.: $\gamma'_{\alpha\beta} = e^{2\omega} \gamma_{\alpha\beta}$

Conformal symmetry:

vanishing stress-energy tensor: $\mathcal{T}_{\alpha\beta} = 0$
traceless stress-energy tensor: $\text{tr } \mathcal{T} = 0$
conformal gauge: $\gamma_{\alpha\beta} = e^\Phi \eta_{\alpha\beta}$

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Conformal properties fixed by **OPEs**.

Action Principle and Conformal Symmetry

Superstrings in D dimensions $\longrightarrow$ *Virasoro algebra* (central extension of de Witt's algebra):

$$\mathcal{T}(z) = -\frac{1}{\alpha'} \partial X(z) \cdot \partial X(z) - \frac{1}{2} \psi(z) \cdot \partial \psi(z) \quad \Rightarrow \quad c = \frac{3}{2} D$$

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$(\lambda, 0) / (1 - \lambda, 0)$ Ghost System

Introduce anti-commuting (b, c) and commuting (β, γ) conformal fields:

$$S_{\text{ghost}}[b, c, \beta, \gamma] = \frac{1}{2\pi} \iint dz d\bar{z} (b(z) \bar{\partial} c(z) + \beta(z) \bar{\partial} \gamma(z))$$

where $\lambda_b = 2$ and $\lambda_c = -1$, and $\lambda_\beta = \frac{3}{2}$ and $\lambda_\gamma = -\frac{1}{2}$.

[Friedan, Martinec, Shenker (1986)]

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Consequence:

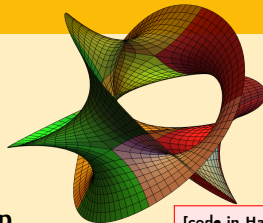
$$c_{\text{full}} = c + c_{\text{ghost}} = 0 \quad \Leftrightarrow \quad D = 10.$$

Extra Dimensions and Compactification

Compactification

$$\mathcal{M}^{1,9} = \mathcal{M}^{1,3} \otimes \mathcal{X}_6$$

- $\mathcal{X}_6$ is a **compact** manifold
- $N = 1$ **supersymmetry** preserved in 4D
- algebra of $SU(3) \otimes SU(2) \otimes U(1)$ in arising **gauge group**



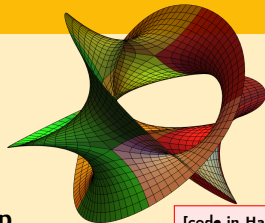
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Kähler manifolds (M, g) such that:

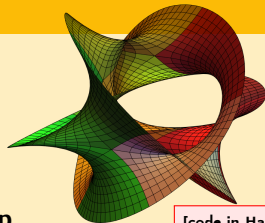
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Characterised by **Hodge numbers**

$$h^{r,s} = \dim_{\mathbb{C}} H_{\bar{\partial}}^{r,s}(M, \mathbb{C})$$

counting the no. of harmonic (r, s) -forms.

D-branes and Open Strings

Polyakov's action naturally introduces **Neumann b.c.** for **open strings**:

$$\partial_\sigma X(\tau, \sigma) \Big|_{\sigma=0}^{\sigma=\ell} = 0$$

satisfied by **open and closed strings** in D dim. s.t. $\square X = 0 \Rightarrow X(z, \bar{z}) = X(z) + \bar{X}(\bar{z})$.

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T-duality

Dirichlet b.c. consequence of **T-duality** on p directions:

$$\bar{X}(z) \mapsto -\bar{X}(z) \quad \Rightarrow \quad \partial_\sigma X^i(\tau, \sigma) \Big|_{\sigma=0}^{\sigma=\ell} = 0 \quad \xrightarrow{T\text{-duality}} \quad \partial_\tau \tilde{X}^i(\tau, \sigma) \Big|_{\sigma=0}^{\sigma=\ell} = 0$$

thus **open strings** can be **constrained** to $D(D - p - 1)$ -branes.

[Polchinski (1995, 1996)]

D-branes and Open Strings

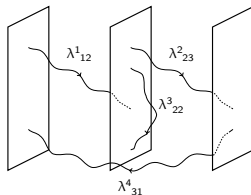
Introducing Dp -branes breaks $\text{ISO}(1, D-1) \rightarrow \text{ISO}(1, p) \otimes \text{SO}(D-1-p)$:

$$\mathcal{A}^\mu \rightarrow (\mathcal{A}^A, \mathcal{A}^a) \quad \Rightarrow \quad \text{U}(1) \text{ theory in } p+1 \text{ dimensions (and scalars)}$$

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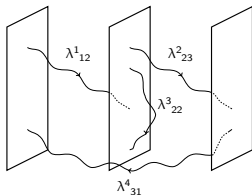
[Chan, Paton (1969)]

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Chan-Paton Factors

When branes are **coincident**:

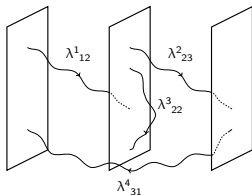
$$\bigoplus_{r=1}^N \text{U}_r(1) \longrightarrow \text{U}(N)$$

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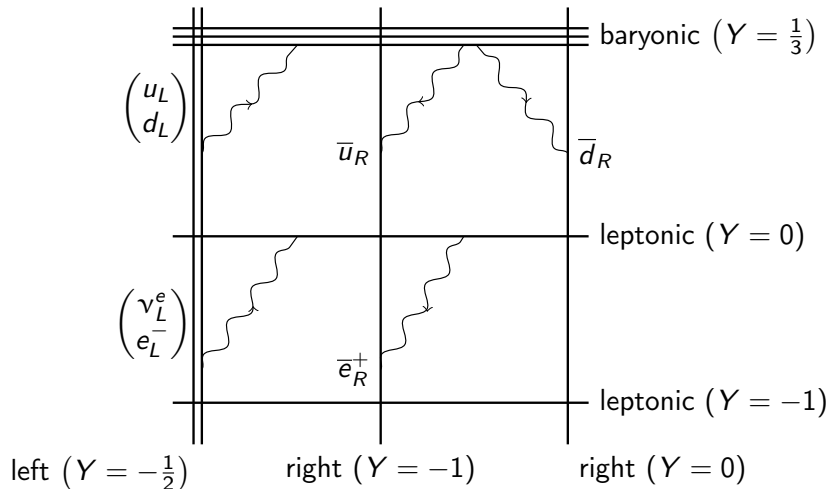
Chan-Paton Factors

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$$\bigoplus_{r=1}^N \text{U}_r(1) \longrightarrow \text{U}(N)$$

Build gauge bosons, fermions and scalars.

Standard Model-like Scenarios



[Zwiebach (2009)]

Intersecting D-branes

Consider N intersecting $D6$ -branes filling $\mathcal{M}^{1,3}$ and embedded in $\mathbb{R}^6$

Twist Fields Correlators

$$\left\langle \prod_{t=1}^{N_B} \sigma_{M_{(t)}}(x_{(t)}) \right\rangle = \mathcal{N} \left(\{x_{(t)}, M_{(t)}\}_{1 \leq t \leq N_B} \right) e^{-S_{E(\text{cl})} \left(\{x_{(t)}, M_{(t)}\}_{1 \leq t \leq N_B} \right)}$$

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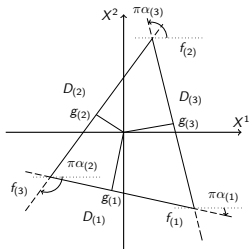
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D-branes in **factorised** internal space:

- **embedded as lines** in $\mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2$
- **relative rotations** are $SO(2) \simeq U(1)$ elements
- $S_E(\text{cl}) \left(\{x(t), M(t)\}_{1 \leq t \leq N_B} \right) \sim \text{Area} \left(\{f(t), R(t)\}_{1 \leq t \leq N_B} \right)$

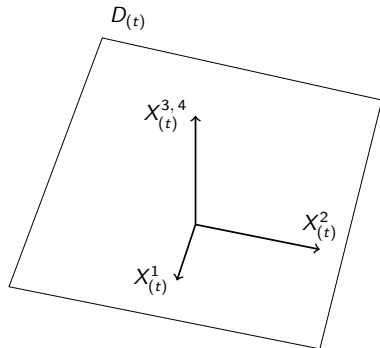
[Cremades, Ibanez, Marchesano (2003); Pesando (2012)]

SO(4) Rotations

Consider $\mathbb{R}^4 \times \mathbb{R}^2$ (focus on $\mathbb{R}^4$):

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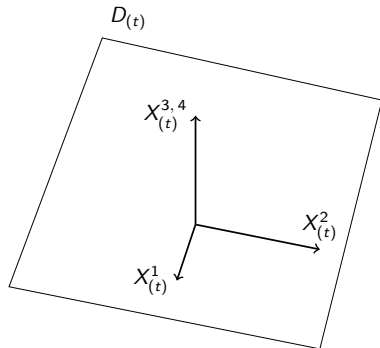
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$$(X_{(t)})^I = (R_{(t)})^I_J X^J - g_{(t)}^I \in \mathbb{R}^4$$

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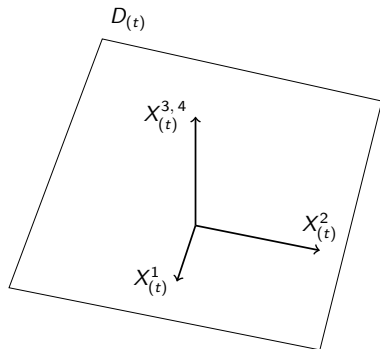
where

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that is

$$[R_{(t)}] = \{R_{(t)} \sim \mathcal{O}_{(t)} R_{(t)}\}$$

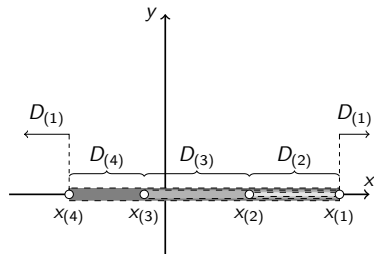
Boundary Conditions

What are the consequences for open strings?

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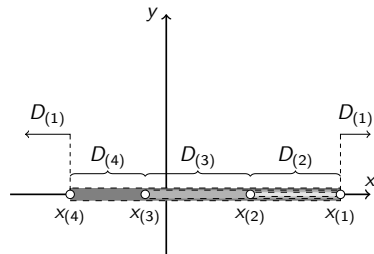
- $u = x + iy = e^{\tau_e + i\sigma}$ and $\bar{u} = u^*$
- $x_{(t)} < x_{(t-1)}$ **worldsheet intersection points**
- $X_{(t)}^{1,2}$ are **Neumann**, $X_{(t)}^{3,4}$ are **Dirichlet**



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Branch Cuts and Discontinuities for $x \in D_{(t)}$

$$\begin{cases} \partial_u X(x + i0^+) = U_{(t)} \cdot \partial_{\bar{u}} \bar{X}(x - i0^+) = \left[R_{(t)}^{-1} \cdot (\sigma_3 \otimes \mathbb{1}_2) \cdot R_{(t)} \right] \cdot \partial_{\bar{u}} \bar{X}(x - i0^+) \\ X(x_{(t)}, x_{(t)}) = f_{(t)} \end{cases}$$

Doubling Trick and Spinor Representation

Doubling Trick

$$\partial_z \mathcal{X}(z) = \begin{cases} \partial_u X(u) & \text{if } z \in \mathcal{H}_{>}^{(\bar{t})} \\ U_{(\bar{t})} \partial_{\bar{u}} \bar{X}(\bar{u}) & \text{if } z \in \mathcal{H}_{<}^{(\bar{t})} \end{cases} \Rightarrow \begin{aligned} \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i} \delta_+) &= \mathcal{U}_{(t, t+1)} \partial_z \mathcal{X}(x_{(t)} + \delta_+), \\ \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i} \delta_-) &= \tilde{\mathcal{U}}_{(t, t+1)} \partial_z \mathcal{X}(x_{(t)} + \delta_-), \end{aligned}$$

where $\mathcal{H}_{\gtrless}^{(t)} = \{z \in \mathbb{C} \mid \text{Im } z \gtrless 0 \text{ or } z \in D_{(t)}\}$ and $\delta_{\pm} = \eta \pm i0^+$.

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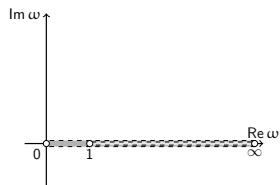
Use Pauli matrices $\tau = (i \mathbb{1}_2, \vec{\sigma})$:

$$\partial_z \mathcal{X}_{(s)}(z) = \partial_z \mathcal{X}'(z) \tau_I \Rightarrow \partial_z \mathcal{X}(x_{(t)} + e^{2\pi i} \delta_{\pm}) = \overset{(\sim)}{\mathcal{L}}_{(t, t+1)} \partial_z \mathcal{X}(x_{(t)} + \delta_{\pm}) \overset{(\sim)}{\mathcal{R}}_{(t, t+1)}$$

where

$$\overset{(\sim)}{\mathcal{L}}_{(t, t+1)} \in \text{SU}(2)_L \quad \text{and} \quad \overset{(\sim)}{\mathcal{R}}_{(t, t+1)} \in \text{SU}(2)_R$$

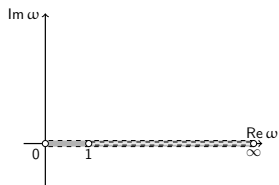
Hypergeometric Basis



Sum over all contributions:

$$\partial_z \mathcal{X}(z) = \sum_{l, r=-\infty}^{+\infty} c_{lr} (-\omega_z)^{A_{lr}} (1 - \omega_z)^{B_{lr}} B_{0,l}^{(L)}(\omega_z) \left(B_{0,r}^{(R)}(\omega_z) \right)^T$$

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Basis of Solutions

$$B_{0,n}(\omega_z) = \begin{pmatrix} 1 & 0 \\ 0 & K_n \end{pmatrix} \begin{pmatrix} \frac{1}{\Gamma(c_n)} {}_2F_1(a_n, b_n; c_n; \omega_z) \\ \frac{(-\omega_z)^{1-c_n}}{\Gamma(2-c_n)} {}_2F_1(a_n + 1 - c_n, b_n + 1 - c_n; 2 - c_n; \omega_z) \end{pmatrix}$$

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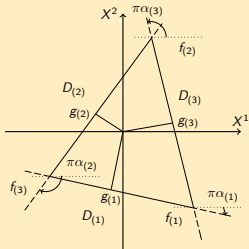
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Physical Interpretation



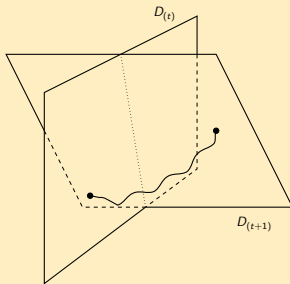
$$\begin{aligned}
 2\pi\alpha' S_{\mathbb{R}^4} \Big|_{\text{on-shell}} &= \sum_{t=1}^3 \left(\frac{1}{2} \left| g_{(t)}^\perp \right| \left| f_{(t-1)} - f_{(t)} \right| \right) \\
 &= \text{Area} \left(\{ f_{(t)} \}_{1 \leq t \leq N_B} \right)
 \end{aligned}$$

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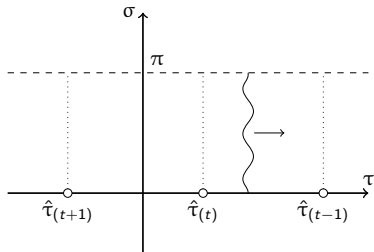
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Physical Interpretation



- strings no longer confined to plane
- strings form a *small bump* from the D-brane
- classical action **larger** than factorised case

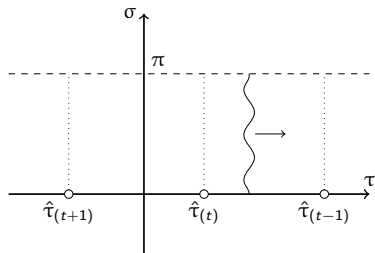
Fermions on the Strip



Action of Boundary Changing Operators

$$\begin{cases} \psi_-^i(\tau, 0) &= (R_{(t)})^I{}_J \psi_+^J(\tau, 0) \quad \text{for } \tau \in (\hat{\tau}_{(t)}, \hat{\tau}_{(t-1)}) \\ \psi_-^I(\tau, \pi) &= -\psi_+^I(\tau, \pi) \quad \text{for } \tau \in \mathbb{R} \end{cases}$$

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Stress-energy Tensor

$$\mathcal{T}_{\pm\pm}(\xi_{\pm}) = -i \frac{T}{4} \psi_{\pm, I}^*(\xi_{\pm}) \overset{\leftrightarrow}{\partial} \psi_{\pm}^I(\xi_{\pm}) \quad \Rightarrow \quad \begin{cases} \dot{H}(\tau) &= 0 \\ \dot{P}(\tau) &\neq 0 \end{cases} \Leftrightarrow \tau \in (\tau_{(t)}, \tau_{(t-1)})$$

Conserved Product and Operators

Expand on a **basis of solutions**

$$\psi_{\pm}(\xi_{\pm}) = \sum_{n=-\infty}^{+\infty} b_n \psi_n(\xi_{\pm}) \quad \Rightarrow \quad \Psi(z) = \begin{cases} \psi_{E,+}(u) & \text{if } z \in \mathcal{H}_{>}^{(\bar{t})} \\ \psi_{E,-}(u) & \text{if } z \in \mathcal{H}_{<}^{(\bar{t})} \end{cases}$$

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$$\langle\langle {}^*\psi_n, \psi_m \rangle\rangle = 2\pi\mathcal{N} \oint \frac{dz}{2\pi i} {}^*\Psi_n {}^*\Psi_m = \delta_{n,m} \quad \Rightarrow \quad \langle\langle {}^*\Psi_n^{(*)}, \Psi^{(*)} \rangle\rangle = b_n^{(\dagger)}$$

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Derive the **algebra of operators**:

$$[b_n, b_m^{\dagger}]_+ = \frac{2\mathcal{N}}{T} \langle\langle {}^*\Psi_n^*, \Psi_m^* \rangle\rangle$$

Twisted Complex Fermions

Consider the case $R_{(t)} = e^{i\pi\alpha_{(t)}} \in U(1)$:

$$\Psi(x_{(t)} + e^{2\pi i}\delta) = e^{i\pi\epsilon_{(t)}} \Psi(x_{(t)} + \delta)$$

where

$$\epsilon_{(t)} = \alpha_{(t+1)} - \alpha_{(t)} + \theta(\alpha_{(t)} - \alpha_{(t+1)} - 1) - \theta(\alpha_{(t+1)} - \alpha_{(t)} - 1)$$

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Basis of Solutions

$$\Psi_n(z; \{x_{(t)}\}) = \mathcal{N}_\Psi z^{-n} \prod_{t=1}^N \left(1 - \frac{z}{x_{(t)}}\right)^{n_{(t)} + \frac{\epsilon_{(t)}}{2}}$$

$$^*\Psi_n(z; \{x_{(t)}\}) = \frac{1}{2\pi\mathcal{N}\mathcal{N}_\Psi} z^{n-1} \prod_{t=1}^N \left(1 - \frac{z}{x_{(t)}}\right)^{-\tilde{n}_{(t)} + \frac{\epsilon_{(t)}}{2}}$$

Vacua

Define the **vacuum** with respect to b_n :

$$b_n \left| \{x_{(t)}\} \right\rangle = 0 \quad \text{for } n \geq 1$$

$$b_n \left| \tilde{0} \right\rangle = 0 \quad \text{for } n \geq n_{(t)} + \frac{\epsilon_{(t)}}{2} + \frac{1}{2}$$

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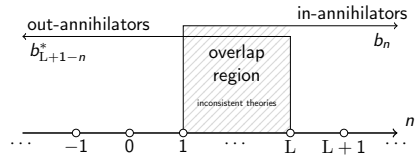
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Theories are subject to **consistency conditions**:

$$L = n_{(t)} + \tilde{n}_{(t)}$$



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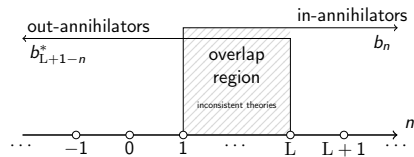
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Stress-energy Tensor and CFT Approach

Compute the OPEs leading to the **stress-energy tensor**:

$$\mathcal{T}(z) = \frac{\pi T}{2} \mathcal{N}_{\Psi}^2 \sum_{n, m=-\infty}^{+\infty} : b_n b_m^* : z^{-n-m} \left[\frac{m-n}{2} + 2 \sum_{t=1}^N \frac{n_{(t)} + \frac{\epsilon_{(t)}}{2}}{z - x_{(t)}} \right] + \frac{1}{2} \left(\sum_{t=1}^N \frac{n_{(t)} + \frac{\epsilon_{(t)}}{2}}{z - x_{(t)}} \right)^2$$

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Invariant Vacuum and Spin Fields

$$|\{x_{(t)}\}\rangle = \mathcal{N}(\{x_{(t)}\}) \mathcal{R} \left[\prod_{t=1}^M \mathcal{S}_{(t)}(x_{(t)}) \right] |0\rangle_{\text{SL}_2(\mathbb{R})}$$

Spin Fields Amplitudes

Equivalence with Bosonization

$$\partial_{x_{(t)}} \langle \{x_{(t)}\} | \{x_{(t)}\} \rangle = \oint_{x_{(t)}} \frac{dz}{2\pi i} \frac{\langle \{x_{(t)}\} | \mathcal{T}(z) | \{x_{(t)}\} \rangle}{\langle \{x_{(t)}\} | \{x_{(t)}\} \rangle}$$

$$\Rightarrow \langle \{x_{(t)}\} | \{x_{(t)}\} \rangle = \mathcal{N}(\{\epsilon_{(t)}\}) \prod_{\substack{t=1 \\ t>u}}^N (x_{(u)} - x_{(t)})^{\left(n_{(u)} + \frac{\epsilon_{(u)}}{2}\right) \left(n_{(t)} + \frac{\epsilon_{(t)}}{2}\right)}$$

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- alternative framework for amplitudes (extension to (non) Abelian twist/spin fields?)

Contents

Conformal Symmetry and Geometry of the Worldsheet

Preliminary Concepts and Tools

D-branes Intersecting at Angles

Fermions and Point-like Defect CFT

Cosmological Backgrounds and Divergences

Orbifolds and Cosmological Toy Models

Null Boost Orbifold

Deep Learning the Geometry of String Theory

Machine Learning and Deep Learning

Machine Learning for String Theory

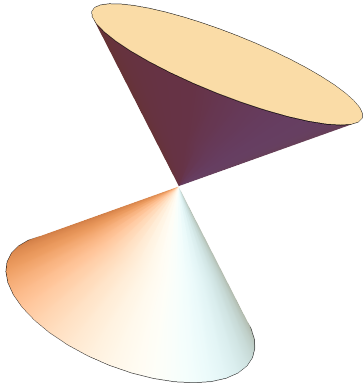
AI Implementations for Geometry and Strings

A Few Words on a Theory of Everything

string theory = theory of everything = nuclear forces + gravity

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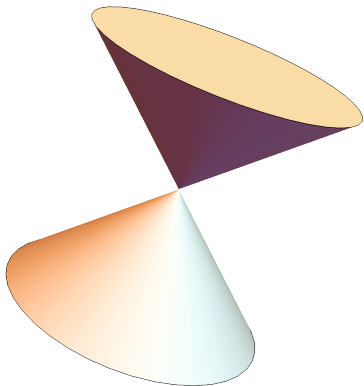


From the phenomenological point of view:

- cosmological implications

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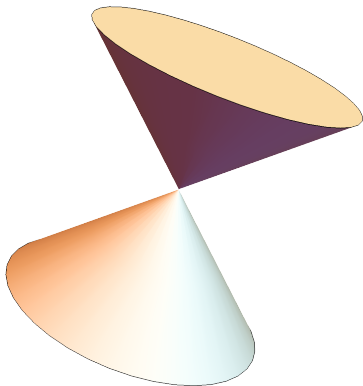


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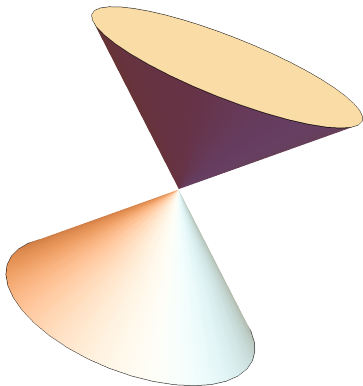


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time-dependent orbifold models

[Craps, Kutasov, Rajesh (2002); Liu, Moore, Seiberg (2002)]

Orbifolds

Mathematics

manifold M

(Lie) group G

stabilizer: $G_p = \{g \in G \mid gp = p \in M\}$

orbit: $Gp = \{gp \in M \mid g \in G\}$

charts $\phi = \pi \circ \mathcal{P}$ where:

$\mathcal{P}: U \subset \mathbb{R}^n \rightarrow U/G$

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additional d.o.f. (*twisted states*)

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Use **time-dependent orbifolds** to model singularities in time

Cosmological Singularities

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divergent closed string amplitudes $\Rightarrow$ gravitational backreaction?

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Divergences

Even in simple models (e.g. NBO, more on this later) the 4 tachyons amplitude is divergent **at tree level**:

$$A_4 \sim \int_{q \sim \infty} \frac{dq}{|q|} \mathcal{A}(q)$$

where

$$\mathcal{A}_{\text{closed}}(q) \sim q^{4-\alpha' \|\vec{p}_\perp\|^2} \quad \text{and} \quad \mathcal{A}_{\text{open}}(q) \sim q^{1-\alpha' \|\vec{p}_\perp\|^2} \text{tr}([T_1, T_2]_+ [T_3, T_4]_+)$$

Null Boost Orbifold

Start from $(x^+, x^-, x^2, \vec{x}) \in \mathcal{M}^{1,D-1}$:

$$\begin{cases} u &= x^- \\ z &= \frac{x^2}{\Delta x^-} \\ v &= x^+ - \frac{1}{2} \frac{(x^2)^2}{x^-} \end{cases} \Rightarrow ds^2 = -2 du dv + (\Delta u)^2 dz^2 + \delta_{ij} dx^i dx^j$$

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Scalars on NBO:

$$\Phi_{\{k_+, l, \vec{k}, r\}}(u, v, z, \vec{x}) = e^{i(k_+ v + l z + \vec{k} \cdot \vec{x})} \tilde{\Phi}_{\{k_+, l, \vec{k}, r\}}(u) = \frac{e^{i(k_+ v + l z + \vec{k} \cdot \vec{x})}}{\sqrt{(2\pi)^D |2\Delta k_+ u|}} e^{-i \frac{l^2}{2\Delta^2 k_+} \frac{1}{u} + i \frac{\|\vec{k}\|^2 + r}{2k_+} u}$$

Scalar QED Interactions

Scalar-photon interactions:

$$S_{\text{sQED}}^{(\text{int})} = \int_{\Omega} d^D x \sqrt{-g} \left(-i e g^{\alpha\beta} a_{\alpha} (\phi^* \partial_{\beta} \phi - \partial_{\beta} \phi^* \phi) + e^2 g^{\alpha\beta} a_{\alpha} a_{\beta} |\phi|^2 - \frac{g^4}{4} |\phi|^4 \right)$$

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Terms involved:

$$\mathcal{I}_{\{N\}}^{[\nu]} = \int_{-\infty}^{+\infty} du |\Delta u| u^{\nu} \prod_{i=1}^N \tilde{\Phi}_{\{k_{+(i)}, l_{(i)}, \vec{k}_{(i)}, r_{(i)}\}}(u)$$

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most terms **do not converge** due to **isolated zeros** ($l_{(*)} \equiv 0$) and cannot be recovered even with a **distributional interpretations** due to the term $\propto u^{-1}$ in the exponential

String and Field Theory

So far:

- field theory presents **divergences** (even sQED $\rightarrow$ eikonal?)

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Massive String States

$$V_M(x; k, S, \xi) =: \left(\frac{i}{\sqrt{2\alpha'}} \xi \cdot \partial_x^2 X(x, x) + \left(\frac{i}{\sqrt{2\alpha'}} \right)^2 S_{\alpha\beta} \partial_x X^\alpha(x, x) \partial_x X^\beta(x, x) \right) e^{ik \cdot X(x, x)}:$$

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*string theory cannot do **better than field theory** (EFT) if the latter **does not exist** (even a Wilson line around z does not prevent such behaviour)*

Resolution and Motivation

Introduce the generalised NBO:

$$\begin{cases} u &= x^- \\ z &= \frac{1}{2x^-} \left(\frac{x^2}{\Delta_2} + \frac{x^3}{\Delta_3} \right) \\ w &= \frac{1}{2x^-} \left(\frac{x^2}{\Delta_2} - \frac{x^3}{\Delta_3} \right) \\ v &= x^+ - \frac{1}{2x^-} \left((x^2)^2 + (x^3)^2 \right) \end{cases}$$

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Distributional Interpretation

$$\tilde{\Phi}_{\{k_+, p, l, \vec{k}, r\}}(u) = \frac{1}{2\sqrt{(2\pi)^D |\Delta_2 \Delta_3 k_+|}} \frac{1}{|u|} e^{-i \left(\frac{1}{8k_+ u} \left[\frac{(l+p)^2}{\Delta_2^2} + \frac{(l-p)^2}{\Delta_3^2} \right] - \frac{\|\vec{k}\|^2 + r}{2k_+} u \right)}$$

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- vanishing volume (**compact orbifold directions**) $\Rightarrow$ particles “cannot escape”
- **non compact** orbifold directions $\Rightarrow$ interpretation of **amplitudes as distributions**
- issue not restricted to NBO/GNBO but also BO, null brane, etc. (it is a **general issues** connected to the geometry of the underlying space)

On the Divergences and Their Nature

- divergences are present in sQED and **open string** sector
- singularities $\Rightarrow$ **massive states** are no longer spectators
- vanishing volume (**compact orbifold directions**) $\Rightarrow$ particles “cannot escape”
- **non compact** orbifold directions $\Rightarrow$ interpretation of **amplitudes as distributions**
- issue not restricted to NBO/GNBO but also BO, null brane, etc. (it is a **general issues** connected to the geometry of the underlying space)

*spacetime singularities are **hidden into contact terms** and interactions with **massive states**
(the gravitational eikonal deals with massless interactions)*

Contents

Conformal Symmetry and Geometry of the Worldsheet

Preliminary Concepts and Tools

D-branes Intersecting at Angles

Fermions and Point-like Defect CFT

Cosmological Backgrounds and Divergences

Orbifolds and Cosmological Toy Models

Null Boost Orbifold

Deep Learning the Geometry of String Theory

Machine Learning and Deep Learning

Machine Learning for String Theory

AI Implementations for Geometry and Strings

The Simplest Calabi–Yau

Focus on Calabi–Yau 3-folds:

$$h^{r,s} = \dim_{\mathbb{C}} H_{\bar{\partial}}^{r,s}(M, \mathbb{C}) \quad \Rightarrow \quad \begin{cases} h^{0,0} & = h^{3,0} = 1 \\ h^{r,0} & = 0 \quad \text{if } r \neq 3 \\ h^{r,s} & = h^{3-r,3-s} \\ h^{1,1}, h^{2,1} \in \mathbb{N} \end{cases}$$

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Complete Intersection Calabi–Yau Manifolds

Intersection of hypersurfaces in

$$\mathcal{A} = \mathbb{P}^{n_1} \times \dots \times \mathbb{P}^{n_m}$$

where

$$\mathbb{P}^n: \quad \begin{cases} p_a(Z^0, \dots, Z^n) & = P_{l_1 \dots l_a} Z^{l_1} \dots Z^{l_a} = 0 \\ p_a(\lambda Z^0, \dots, \lambda Z^n) & = \lambda^a p_a(Z^0, \dots, Z^n) \end{cases}$$

[Green, Hübsch (1987); Hübsch (1992)]

Representation of the Output

CICY can be generalised to m projective spaces and k equations. The problem is thus mapped to:

$$\mathcal{R}: \mathbb{Z}^{m \times k} \longrightarrow \mathbb{N}$$

$$\left[\begin{array}{c|ccc} \mathbb{P}^{n_1} & a_1^1 & \dots & a_k^1 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{P}^{n_m} & a_1^m & \dots & a_k^m \end{array} \right] \longrightarrow h^{1,1} \text{ or } h^{2,1}$$

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Machine Learning Approach

What is $\mathcal{R}$?

$$\mathcal{R}(M) \longrightarrow \mathcal{R}_n(M; w) \quad \text{s.t.} \quad \lim_{n \rightarrow \infty} |\mathcal{R}(M) - \mathcal{R}_n(M; w)| = 0$$

Machine Learning

- exchange **analytical solution** with **optimisation problem**

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Machine Learning

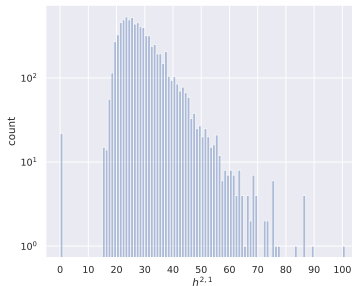
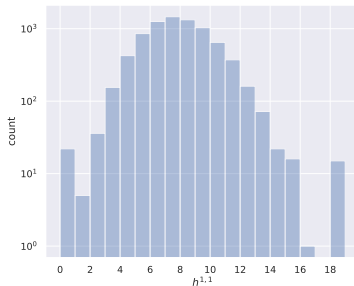
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- provide in-depth **data analysis** of the datasets



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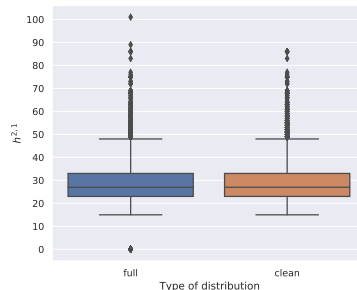
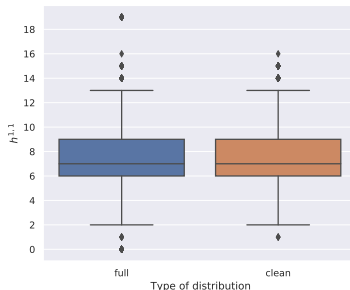
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Exploratory Data Analysis

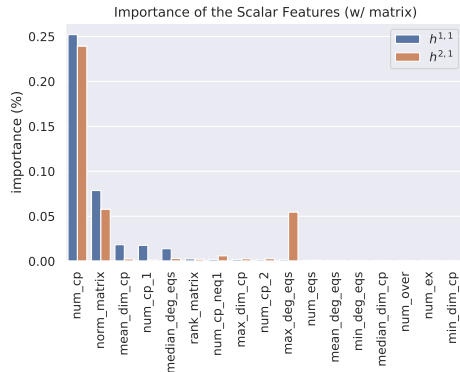
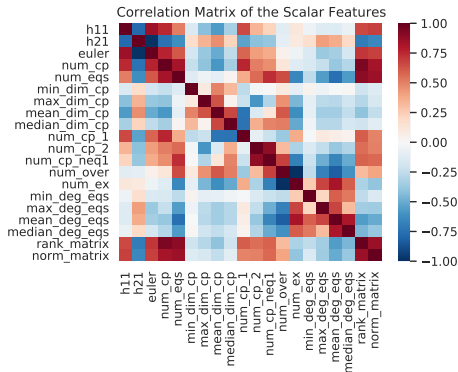
Machine Learning pipeline:

exploratory data analysis → feature **selection** → Hodge numbers

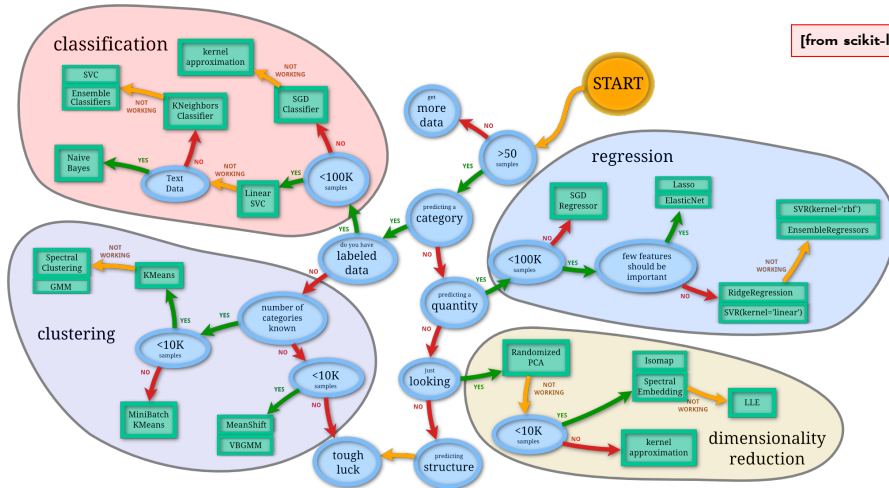
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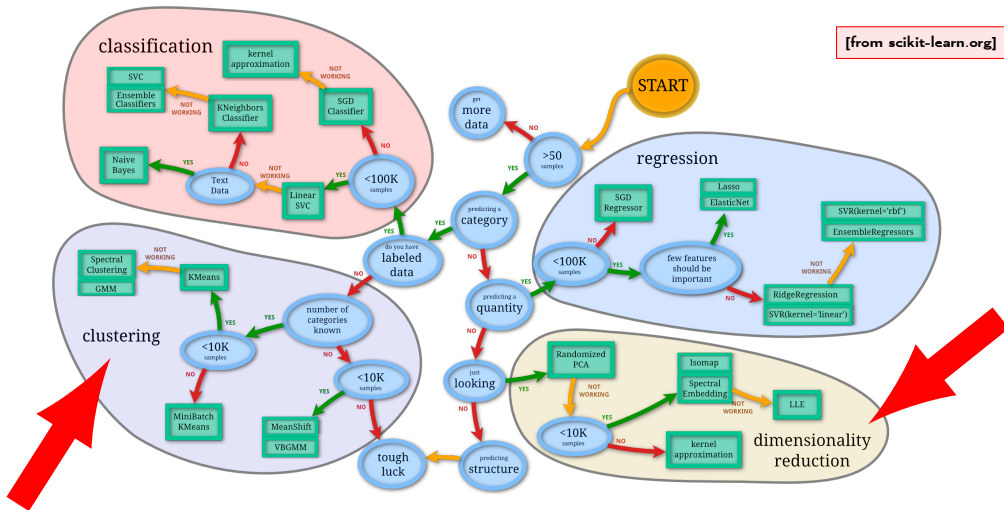
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Machine Learning

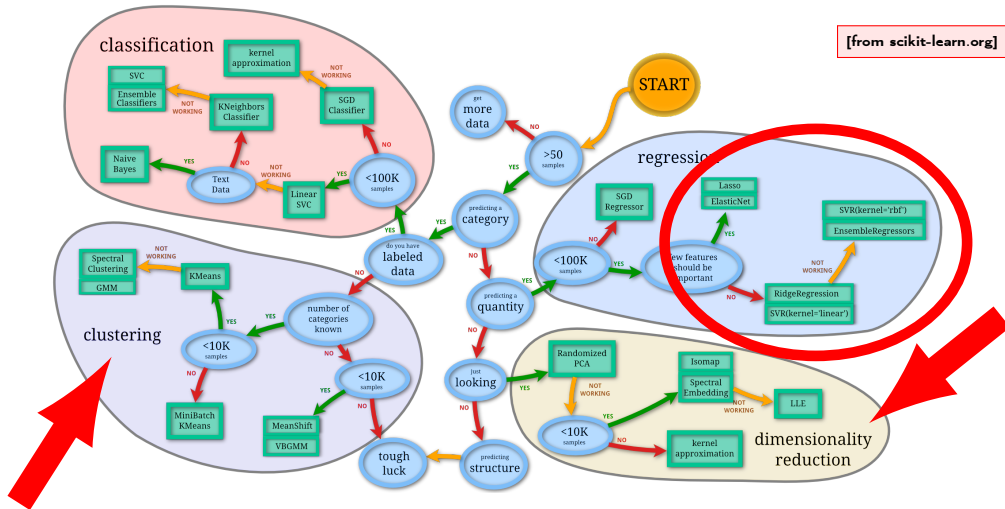


[from scikit-learn.org]



Machine Learning

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A Word on PCA

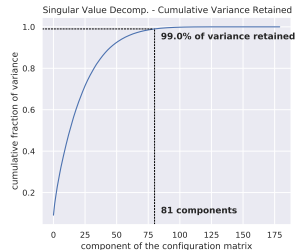
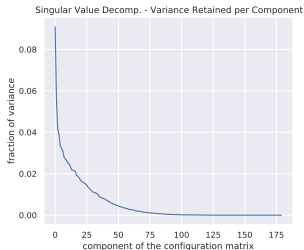
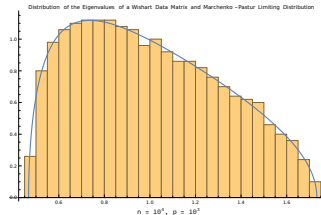
What is PCA for a $X \in \mathbb{R}^{n \times p}$?

- project data onto a **lower dimensional** space where variance is maximised
- equivalently compute the **eigenvectors** of XX^T or the **singular values** of X
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- ease the machine learning job of finding a better representation of the input

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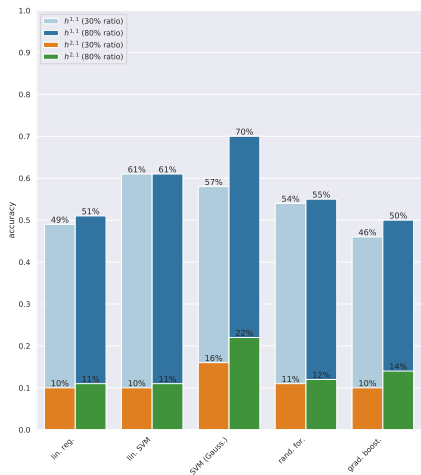
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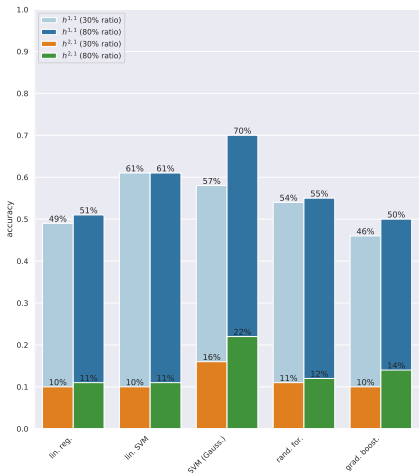
Machine Learning Results

Configuration Matrix Only

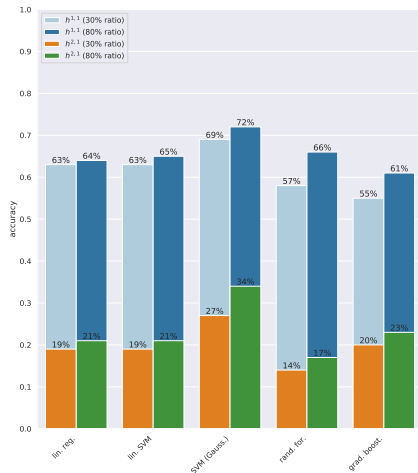


Machine Learning Results

Configuration Matrix Only



Best Training Set



Artificial Intelligence and Neural Networks

- use **gradient descent** to optimise **weights**
- learn highly **non linear** representations of the input
- can be “large” to have enough parameters
- can be “deep” to learn **complicated functions**

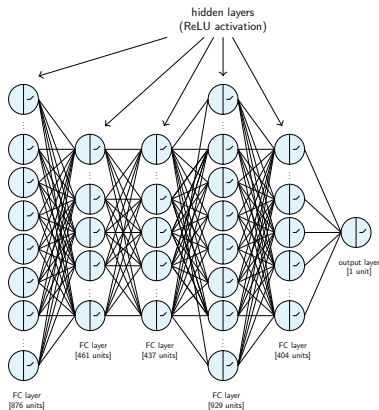
Layers

fully connected: $\phi(a^{(i)\{l\}} \cdot W^{\{l\}} + b^{\{l\}} \mathbb{1}_l)$

convolutional: $\phi(a^{(i)\{l\}} * W^{\{l\}} + b^{\{l\}} \mathbb{1}_l)$

Non linearity ensured by:

$$\phi(z) = \text{ReLU}(z) = \max(0, z)$$

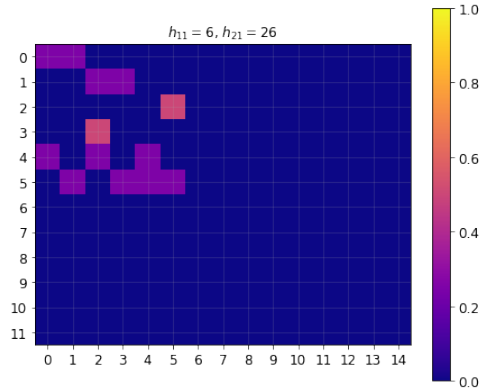


[rendition of the neural network in Bull et al. (2018)]

Convolutional Neural Networks

Why convolutional?

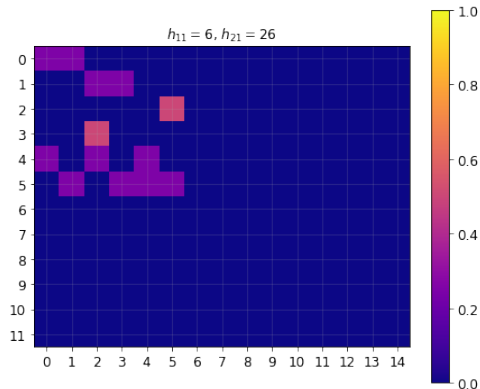
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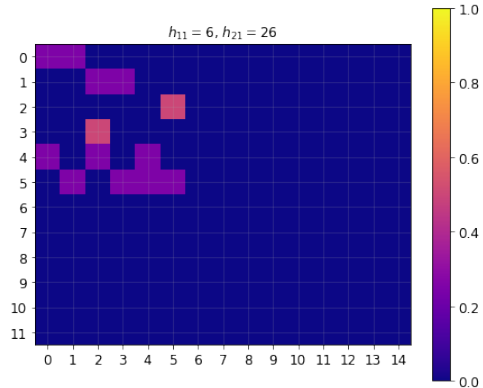
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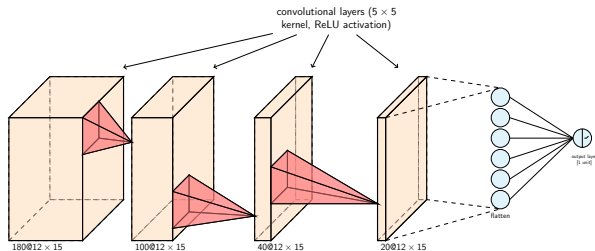
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- find patterns as in **computer vision**



Inception Neural Networks

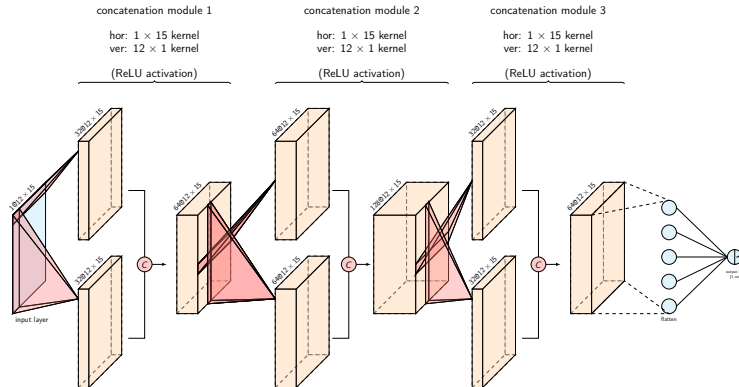
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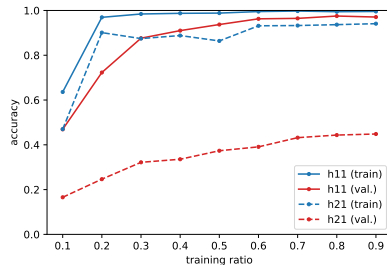
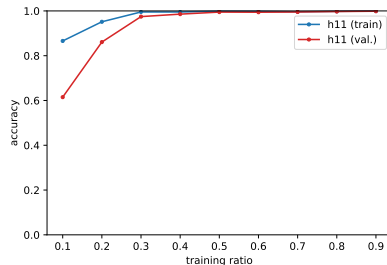
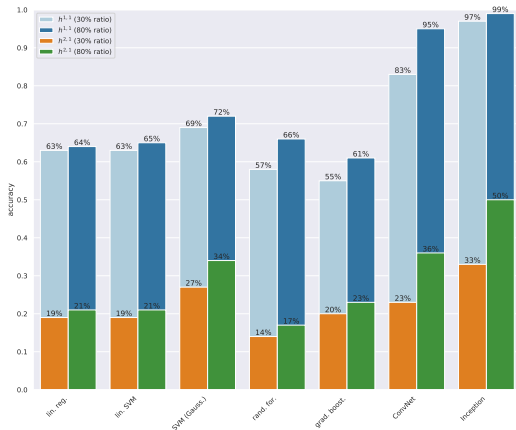
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A Few Comments and Future Directions

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- reinforcement learning $\Rightarrow$ give the rules, not the result!

The End?



THANK YOU!